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Double diffusive convection in a porous layer using a thermal non-equilibrium model

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Abstract

Double diffusive convection in a fluid-saturated porous layer heated from below and cooled from above is studied when the fluid and solid phases are not in local thermal equilibrium, using both linear and nonlinear stability analyses. The Darcy model with time derivative term is employed as momentum equation. A two-field model that represents the fluid and solid phase temperature fields separately is used for energy equation. The onset criterion for stationary, oscillatory and finite amplitude convection is derived analytically. It is found that small inter-phase heat transfer coefficient has significant effect on the stability of the system. There is a competition between the processes of thermal and solute diffusion that causes the convection to set in through either oscillatory or finite amplitude mode rather than stationary. The effect of solute Rayleigh number, porosity modified conductivity ratio, Lewis number, ratio of diffusivities and Vadasz number on the stability of the system is investigated. The nonlinear theory based on the truncated representation of Fourier series method predicts the occurrence of subcritical instability in the form of finite amplitude motions. The effect of thermal non-equilibrium on heat and mass transfer is also brought out.

Keywords: Double diffusive convection; Local thermal non-equilibrium; Porous layer

1. Introduction

The problem of double diffusive convection in porous media has attracted considerable interest during the last few decades because of its wide range of applications, from the solidification of binary mixtures to the migration of solutes in water-saturated soils. The other examples include geophysical systems, electrochemistry, the migration of moisture through air contained in fibrous insulation.

Early studies on the phenomena of double diffusive convection in porous media are mainly concerned with problem of convective instability in a horizontal layer heated and salted from below. A comprehensive review of the literature concerning double diffusive natural convection in a fluid-saturated porous medium may be found in the book by Nield and Bejan [1]. The study of double diffusive convection in porous medium is first under taken by Nield [2] on the basis of linear stability theory for various thermal and solutal boundary conditions. The onset of double diffusive convection in a horizontal porous layer has been investigated by Rudraiah et al. [3] using nonlinear perturbation theory. The linear stability analysis of the thermosolutal convection is carried out by Poulikakos [4] using the Darcy–Brinkman model. The double diffusive convection in porous media in the presence of cross-diffusion effects is analyzed by Rudraiah and Malashetty [5]. The problem of double diffusive convection in a fluid saturated porous layer was later on investigated by many authors (Taunton et al. [6], Taslim and Narusawa [7], Trevisan and Bejan [8], Murray and Chen [9]).

Straughan and Hutter [10] have investigated the double diffusive convection with Soret effect in a porous layer using Darcy–Brinkman model and derived a priori bounds. An analytical and numerical study of double diffusive parallel flow in a horizontal sparsely packed porous layer under the influence of constant heat and mass flux is performed using a Brinkman model by Amahmid et al. [11]. Mamou and Vasseur [12] have

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Nomenclature

a	horizontal wavenumber
c	specific heat
d	depth of the porous layer
D	solute diffusivity
Da	Darcy number K/d^2
$\mathbf{g}$	gravitational acceleration
h	inter-phase heat transfer coefficient
H	non-dimensional inter-phase heat transfer coefficient, $hd^2/\varepsilon k_f$
$\mathbf{k}$	unit vector in the vertical direction,
k	thermal conductivity
K	permeability of the porous layer
l, m	wavenumbers in x-, y-directions
Le	Lewis number, κ_f/D
Nu	Nusselt number,
Pr	Prandtl number, ν/κ_f
p	pressure
$\mathbf{q}$	velocity vector, (u, v, w)
Ra_T	thermal Rayleigh number, $\beta_T g \Delta T d K/\varepsilon \nu \kappa_f$
Ra_S	solute Rayleigh number, $\beta_S g \Delta S d K/\varepsilon \nu \kappa_f$
S	solute concentration
Sh	Sherwood number
T	temperature,
t	time
Va	Vadasz number, $\varepsilon \frac{Pr}{Da}$
x, y, z	space coordinates

Greek symbols

α	the ratio of diffusivities, $(\rho_0 c)_s k_f/(\rho_0 c)_f k_s = \kappa_f/\kappa_s$
β_S	solute expansion coefficient
β_T	thermal expansion coefficient
γ	the porosity modified conductivity ratio, $\varepsilon k_f/(1-\varepsilon)k_s$
δ^2	$\pi^2 + a^2$
ε	porosity
κ	thermal diffusivity, $k/(\rho_0 c)$
μ	dynamic viscosity
ν	kinematic viscosity, μ/ρ_0
ρ	fluid density
ω	frequency
ψ	stream function

Subscripts/superscripts

b	basic state
c	critical
f	fluid phase
F	finite amplitude
l	lower wall
Osc	oscillatory
s	solid phase
St	stationary
u	upper wall
0	reference
*	non-dimensional
′	perturbed quantity

studied the double diffusive instability in a horizontal rectangular porous enclosure subject to vertical temperature and concentration gradients Double diffusive convection in a vertical enclosure filled with anisotropic porous media has been studied numerically by Bennacer et al. [13]. Mamou et al. [14] performed both analytical and numerical stability analyses of double diffusive convection in a confined horizontal rectangular enclosure based on Galerkin and finite element methods respectively. Using the Darcy–Brinkman model Bennacer et al. [15] have studied a thermosolutal convection in a two dimensional rectangular cavity filled with saturated homogeneous porous medium that is thermally anisotropic. They have presented an analytical and numerical study of combined heat and mass transfer driven by buoyancy, due to temperature and concentration variation. Bahloul et al. [16] have carried out an analytical and numerical study of the double diffusive convection in a shallow horizontal porous layer under the influence of Soret effect.

Recently, Hill [17] performed linear and nonlinear stability analyses of double diffusive convection in a fluid saturated porous layer with a concentration based internal heat source using Darcy's law. Double diffusive natural convection within a multilayer anisotropic porous medium is studied numerically and analytically by Bennacer et al. [18]. Mansour et al. [19] have investigated the multiplicity of solutions induced by thermosolutal convection in a square porous cavity heated from below and subjected to horizontal concentration gradient in the presence of Soret effect.

In modeling a fluid-saturated porous medium all the above investigations on double diffusive convection have assumed a state of local thermal equilibrium (LTE) between the fluid and the solid phase at any point in the medium. This is a common practice for most of the studies where the temperature gradient at any location between the two phases is assumed to be negligible. For many practical applications, involving high-speed flows or large temperature differences between the fluid and solid phases, the assumption of local thermal equilibrium is inadequate and it is important to take account of the thermal non-equilibrium effects. Due to applications of porous media theory in drying, freezing of foods and other mundane materials and applications in everyday technology such as microwave heating, rapid heat transfer from computer chips via use of porous metal foams and their use in heat pipes, it is believed that local thermal non-equilibrium (LTNE) theory will play a major role in future developments.

Recently, attention has been given to the LTNE model in the study of convection heat transfer in porous media. Much of this work has been reviewed in recent books by Ingham and Pop [20,21] and Nield and Bejan [1]. Criteria for heat and mass transfer models in metal hydride packed beds has been investigated by Kuznetsov and Vafai [22] and effects of non-equilibrium were suggested to be more significant at high Reynolds number and for high porosity. Kuznetsov [23] studied a perturbation solution for a thermal non-equilibrium fluid flow through a three-dimensional sensible storage packed bed. Vafai and Amiri [24] gave detailed information about the work on thermal non-equilibrium effects of fluid flow through a porous packed bed. The review of Kuznetsov [25] gives detailed information about the most but very latest works on thermal non-equilibrium effects on internal forced convection flows. An excellent review of research on local thermal non-equilibrium phenomena in porous medium convection, primarily free and forced convection boundary layers and free convection within cavities, is given by Rees and Pop [26].

Rees and co-workers (Rees and Pop [27,28], Banu and Rees [29]) in a series of studies have investigated thermal non-equilibrium (LTNE) effect on free convective flows in a porous medium. Free convection in a square porous cavity using a thermal non-equilibrium model is studied by Baytas and Pop [30] while Baytas [31] investigated the thermal non-equilibrium natural convection in a square enclosure filled with a heat generating solid phase and with the Brinkman–Forchheimer extended Darcy law for the momentum equation. A review of thermal non-equilibrium free convection in a cavity filled with non-Darcy porous medium is also given by Baytas [32]. The problem of two-dimensional steady mixed convection in a vertical porous layer using thermal non-equilibrium model is investigated numerically by Saeid [33]. The effect of thermal non-equilibrium on the onset of convection in a porous layer using the Lapwood–Brinkman model and also including anisotropy in permeability and thermal diffusivity in a densely packed porous layer have been investigated by Malashetty et al. [34, 35]. More recently, Straughan [36] has considered a problem of thermal convection in a fluid-saturated porous layer using a global nonlinear stability analysis with a thermal non-equilibrium model.

While the double diffusive convection in porous medium is extensively studied for the case of local thermal equilibrium model, there seems to have been no work, to the best knowledge of the authors, on study of double diffusive convection in a fluid-saturated porous layer with thermal non-equilibrium model. In this paper, we intend to perform the linear and a weak nonlinear stability analysis of the double diffusive convection in a fluid-saturated porous layer with the assumption that the fluid and solid phases are not in local thermal equilibrium. Our objective in this paper is to study how the onset criterion for steady, oscillatory and finite amplitude convection and also the heat and mass transfer are affected by the local thermal non-equilibrium model.

2. Mathematical formulation

We consider an infinite horizontal fluid-saturated porous layer confined between the planes $z = 0$ and $z = d$, with the vertically downward gravity force $\mathbf{g}$ acting on it. A uniform adverse temperature gradient $\Delta T = (T_l - T_u)$ and a stabilizing concentration gradient $\Delta S = (S_l - S_u)$ where $T_l > T_u$ and $S_l > S_u$ are maintained between the lower and upper surfaces. A Cartesian frame of reference is chosen with the origin in the lower boundary and the z-axis vertically upwards. The Darcy law with time derivative term is used to model the momentum equation

$$\frac{1}{\varepsilon}\frac{\partial \mathbf{q}}{\partial t} + \frac{\nu}{K}\mathbf{q} = -\frac{1}{\rho_0}\nabla p + \frac{\rho_f}{\rho_0}\mathbf{g} \tag{2.1}$$

where ε and K denote porosity and permeability respectively, ρ and ν are density and kinematic viscosity respectively. The justification for the inclusion of time derivative term in the momentum equation is discussed at the end of this section.

In modeling energy equation for a fluid-saturated porous system, two kinds of theoretical approaches have been used. In the first model, the fluid and solid structures are assumed to be in local thermal equilibrium. This assumption is satisfactory for small-pore media such as geothermal reservoirs and fibrous insulations and small temperature differences between the phases. In the second kind of approach, the fluid and solid structures are assumed to be in thermal non-equilibrium. For many applications involving high-speed flows or large temperature difference between the fluid and solid phases, it is important to take account of the thermal non-equilibrium effects. If the temperatures difference between phases is a very important safety parameter (e.g., fixed bed nuclear propulsion systems and nuclear reactor modeling), the thermal non-equilibrium model in the porous media is an indispensable model.

The local thermal non-equilibrium, which account for the transfer of heat between the fluid and solid phases is considered. A two-field model that represents the fluid and solid phase temperature fields separately, is employed for the energy equation [1]

$$\varepsilon(\rho_0 c)_f \frac{\partial T_f}{\partial t} + (\rho_0 c)_f (\mathbf{q}\cdot\nabla) T_f = \varepsilon k_f \nabla^2 T_f + h(T_s - T_f) \tag{2.2}$$

$$(1-\varepsilon)(\rho_0 c)_s \frac{\partial T_s}{\partial t} = (1-\varepsilon) k_s \nabla^2 T_s - h(T_s - T_f) \tag{2.3}$$

where c is the specific heat, k, the thermal conductivity and h being the inter-phase heat transfer coefficient. In two-field model the energy equations are coupled by means of the terms, which accounts for the heat lost to or gained from the other phase. The inter-phase heat transfer coefficient h depends on the nature of the porous matrix and the saturating fluid and the value of this coefficient has been the subject of intense experimental interest. Large values of h correspond to a rapid transfer of heat between the phases (LTE) and small values of h gives rise to relatively strong thermal non-equilibrium effects. In Eqs. (2.2)–(2.3) T_f and T_s are intrinsic average of the temperature fields and this allows one to set $T_f = T_s = T_w$

whenever the boundary of the porous medium is maintained at the temperature T_w.

The equation of continuity, solute concentration and state are

$$\nabla \cdot \mathbf{q} = 0 \tag{2.4}$$

$$\frac{\partial S}{\partial t} + \frac{1}{\varepsilon}(\mathbf{q}\cdot\nabla)S = D\nabla^2 S \tag{2.5}$$

$$\rho = \rho_0\big[1 - \beta_T(T_f - T_l) + \beta_S(S - S_l)\big] \tag{2.6}$$

where β_T, β_S and D are the thermal and solute expansion coefficient and solute diffusivity respectively.

The basic state is assumed to be quiescent and is given by

$$\mathbf{q}_b = 0, \qquad T_f = T_{fb}(z), \qquad T_s = T_{sb}(z)$$
$$S = S_b(z), \qquad h = 0 \tag{2.7}$$

The basic state temperatures and concentration satisfy the equations

$$\frac{d^2 T_{fb}}{dz^2} = 0, \quad \frac{d^2 T_{sb}}{dz^2} = 0, \quad \frac{d^2 S_b}{dz^2} = 0 \tag{2.8}$$

with boundary conditions

$$T_{fb} = T_{sb} = T_l \quad \text{and} \quad S_b = S_l \quad \text{at } z = 0 \tag{2.9}$$

$$T_{fb} = T_{sb} = T_u \quad \text{and} \quad S_b = S_u \quad \text{at } z = d \tag{2.10}$$

so that the conduction state solutions are given by

$$T_{fb} = T_{sb} = -\frac{\Delta T}{d}z + T_l \tag{2.11}$$

$$S_b = -\frac{\Delta S}{d}z + S_l \tag{2.12}$$

We now superimpose the infinitesimal perturbations on the basic state and study the stability of the system.

Let the basic state be perturbed by an infinitesimal thermal perturbation, so that

$$\mathbf{q} = \mathbf{q}', \qquad T_f = T_{fb} + T_f', \qquad T_s = T_{sb} + T_s'$$
$$S = S_b + S', \qquad p = p_b + p', \qquad \rho = \rho_b + \rho' \tag{2.13}$$

where the prime indicates that the quantities are infinitesimal perturbations. Substituting Eq. (2.13) into Eqs. (2.1)–(2.6) and using the basic state solutions, we obtain the equations governing the perturbations in the form,

$$\frac{1}{\varepsilon}\frac{\partial \mathbf{q}'}{\partial t} + \frac{\nu}{K}\mathbf{q}' = -\frac{1}{\rho_0}\nabla p' + (\beta_S S' - \beta_T T_f')\mathbf{k} \tag{2.14}$$

$$\varepsilon(\rho_0 c)_f\frac{\partial T_f'}{\partial t} + (\rho_0 c)_f(\mathbf{q}'\cdot\nabla)T_f' + (\rho_0 c)_f w'\left(\frac{dT_{fb}}{dz}\right)$$
$$= \varepsilon k_f \nabla^2 T_f' + h(T_s' - T_f') \tag{2.15}$$

$$(1-\varepsilon)(\rho_0 c)_s\frac{\partial T_s'}{\partial t} = (1-\varepsilon)k_s\nabla^2 T_s' - h(T_s' - T_f') \tag{2.16}$$

$$\frac{\partial S'}{\partial t} + \frac{1}{\varepsilon}(\mathbf{q}'\cdot\nabla)S' + \frac{1}{\varepsilon}w'\left(\frac{dS_b}{dz}\right) = D\nabla^2 S' \tag{2.17}$$

We consider only two-dimensional perturbations by ignoring the variations in y-direction. By operating curl twice on Eq. (2.14) we eliminate p' from it, and then render the resulting equation and Eqs. (2.15)–(2.17) dimensionless using the following transformations

$$(x, y, z) = (x^*, y^*, z^*)d, \qquad t = \frac{(\rho_0 c)_f d^2}{k_f}t^*$$

$$(u', v', w') = \frac{\varepsilon k_f}{(\rho_0 c)_f d}(u^*, v^*, w^*)$$

$$T_f' = (\Delta T)T_f^*, \qquad T_s' = (\Delta T)T_s^*, \qquad S' = (\Delta S)S^* \tag{2.18}$$

to obtain non-dimensional equations as (on dropping the asterisks for simplicity),

$$\left(\frac{1}{Va}\frac{\partial}{\partial t} + 1\right)\nabla^2\psi + Ra_T\frac{\partial T_f}{\partial x} - Ra_S\frac{\partial S}{\partial x} = 0 \tag{2.19}$$

$$\left(\frac{\partial}{\partial t} - \nabla^2\right)T_f - \frac{\partial(\psi, T_f)}{\partial(x, z)} + \frac{\partial\psi}{\partial x} = H(T_s - T_f) \tag{2.20}$$

$$\left(\alpha\frac{\partial}{\partial t} - \nabla^2\right)T_s = \gamma H(T_f - T_s) \tag{2.21}$$

$$\left(\frac{\partial}{\partial t} - \frac{1}{Le}\nabla^2\right)S - \frac{\partial(\psi, S)}{\partial(x, z)} + \frac{\partial\psi}{\partial x} = 0 \tag{2.22}$$

where

$Va = \varepsilon\frac{Pr}{Da}$, the Vadasz number,
$Ra_T = \beta_T g\Delta T dK/\varepsilon\nu\kappa_f$, the thermal Rayleigh number,
$Ra_S = \beta_S g\Delta S dK/\varepsilon\nu\kappa_f$, the solute Rayleigh number,
$H = hd^2/\varepsilon k_f$, the inter-phase heat transfer coefficient,
$\alpha = \kappa_f/\kappa_s$, the ratio of diffusivities,
$\gamma = \varepsilon k_f/(1-\varepsilon)k_s$, the porosity modified conductivity ratio,
$Le = \kappa_f/D$, the Lewis number

with $\kappa_f = k_f/(\rho_0 c)_f$ being the thermal diffusivity and ψ being the stream function defined such that $u = \partial\psi/\partial z$, $w = -\partial\psi/\partial x$. It should be noted that the dimensionless group H is akin to the Nield number. Vadasz [37] in his work, defined the Nield number as $Ni = (1-\varepsilon)k_s/(hd^2)$. Therefore, the dimensionless group H used in the present paper may be defined as $H = 1/Ni_f$ where Ni_f is the fluid related Nield number given by $Ni_f = \varepsilon k_f/(hd^2)$. It is worth mentioning that the Rayleigh number Ra_T defined above is based on the properties of the fluid while

$$Ra_{T_{\mathrm{LTE}}} = \left(\frac{\gamma}{1+\gamma}\right)Ra_T = \frac{\rho_f g\beta_T(\Delta T)Kd}{[\varepsilon k_f + (1-\varepsilon)k_s]} \tag{2.23}$$

is the Rayleigh number based on the mean properties of the porous medium and it is this latter definition, which is used in the thermal equilibrium model. Vadasz [38] was the first to show the substantial impact of the new dimensionless group $Va = \varepsilon\frac{Pr}{Da}$, which includes the Prandtl number, the Darcy number and the porosity of the porous medium, on convection in porous media and was the first to identify the conditions for such impact. Straughan [39] named this dimensionless group as Vadasz number. Vadasz in a series of comprehensive works [38, 40–42] reported that the typical values of Vadasz number (Va) in traditional porous media applications are quite big, a fact which provides the justification for neglecting the time derivative term in Eq. (2.19). This is then the classical theory of Darcy. Nield and Bejan [1] argue for this scenario, which is certainly true in many geophysical and engineering applications.

However, Vadasz [38] argues that in circumstances linked to modern porous media applications the value of *Va* can become of unit order of magnitude or even smaller, in which case the time derivative should be retained. Straughan [39] has also supported this argument. Accordingly, following Vadasz [38] line of argument, in the present paper, we keep the time derivative terms in the Darcy equation and we will find how Vadasz number *Va* influences the overstable motions. Including the time derivative term in Eq. (2.19) is equivalent to maintaining the highest derivative in an equation to satisfy all boundary (initial) conditions. Further it is important to note that it is only through this combined dimensionless group that the Prandtl number affects the flow in the porous media. One can refer Vadasz [38] for a full discussion on the numerical values that the Prandtl number can assume in a typical porous medium.

Since the fluid and solid phases are not in thermal equilibrium, the use of appropriate thermal boundary conditions may pose a difficulty. However, the assumption made earlier that the solid and fluid phases share the same temperature as that of the boundary temperatures helps in overcoming this difficulty. Accordingly, Eqs. (2.19)–(2.22) are solved for impermeable isothermal isosolutal boundaries. Hence the boundary conditions for the perturbation variables are given by

$$\psi = T_f = T_s = S = 0 \quad \text{at } z = 0, 1 \tag{2.24}$$

3. Linear stability analysis

In this section we predict the thresholds of both marginal and oscillatory convections using linear theory. The eigenvalue problem defined by Eqs. (2.19)–(2.22) subject to the boundary conditions (2.24) is solved using the time-dependent periodic disturbances in a horizontal plane, upon assuming that amplitudes are small enough and can be expressed as

$$\begin{pmatrix} \psi \\ T_f \\ T_s \\ S \end{pmatrix} = e^{i\omega t} \begin{pmatrix} \Psi \sin(ax) \\ \Theta \cos(ax) \\ \Phi \cos(ax) \\ \Sigma \cos(ax) \end{pmatrix} \sin(\pi z) \tag{3.1}$$

where a is a horizontal wavenumbers and ω is the growth rate. Infinitesimal perturbations of the rest state may either damp or grow depending on the value of the parameter ω. Substituting Eqs. (3.1) into Eqs. (2.19)–(2.22) we obtain a matrix equation

$$\begin{pmatrix} \delta^2(\omega/Va + 1) & -a\,Ra_T & 0 & a\,Ra_S \\ -a & \omega + \delta^2 + H & -H & 0 \\ 0 & -\gamma H & \alpha\omega + \delta^2 + \gamma H & 0 \\ -a & 0 & 0 & \omega + \delta^2/Le \end{pmatrix} \times \begin{pmatrix} \Psi \\ \Theta \\ \Phi \\ \Sigma \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{3.2}$$

where

$$\delta^2 = \pi^2 + a^2 \tag{3.3}$$

is the total wavenumber.

For the above matrix equation (3.2) to have the nontrivial solution, we require

$$Ra_T = \frac{\delta^2}{a^2}\big\{\big[\delta^2 + H(1+\gamma)\big] + \omega\big[\alpha\omega + \delta^2(1+\alpha) + H(\alpha+\gamma)\big]\big\}/(\alpha\omega + \delta^2 + \gamma H) \times \left[\delta^2\left(\frac{\omega}{Va} + 1\right) + \frac{a^2 Ra_S}{\omega + \delta^2/Le}\right] \tag{3.4}$$

The growth rate ω is in general a complex quantity such that $\omega = \omega_r + i\omega_i$. The system with $\omega_r < 0$ is always stable, while for $\omega_r > 0$ it will become unstable. For neutral stability state $\omega_r = 0$. Therefore, we now set $\omega = i\omega_i$ in Eq. (3.4) and clear the complex quantities from the denominator, to obtain

$$Ra_T = \Delta_1 + i\omega_i \Delta_2 \tag{3.5}$$

where

$$\Delta_1 = A_0(A_1 + A_2) \tag{3.6}$$

$$\Delta_2 = A_0(A_3 + A_4) \tag{3.7}$$

with

$$A_0 = a^{-2}\big[Va(\delta^4 + Le^2\omega_i^2)\big\{(\delta^2 + H\gamma)^2 + \alpha^2\omega_i^2\big\}\big]^{-1} \tag{3.8}$$

$$A_1 = \delta^2(\delta^4 + Le^2\omega_i^2)\big[\delta^2\,Va(\delta^2 + \gamma H)\big\{\delta^2 + H(\gamma+1)\big\} - \omega_i^2\big\{\delta^4 + (2\gamma H - \alpha^2 Va)\delta^2 + \gamma H^2(\alpha+\gamma) - \alpha^2\,Va\,H\big\}\big] \tag{3.9}$$

$$A_2 = a^2\,Le\,Ra_S\,Va\big[\alpha^2\,Le\,\omega_i^4 + \big\{\delta^4(\alpha^2 + Le) + \delta^2 H(\alpha^2 + 2\gamma\,Le) + \gamma H^2 Le(\alpha+\gamma)\big\}\omega_i^2 + \delta^4(\delta^2 + \gamma H)\big\{\delta^2 + H(\gamma+1)\big\}\big] \tag{3.10}$$

$$A_3 = \delta^2(\delta^4 + Le^2\omega_i^2)\big[\gamma H^2\,Va(\alpha+\gamma) + \alpha^2\omega_i^2(H + Va) + \delta^2\big\{\alpha^2\omega_i^2 + 2\gamma H\,Va + \gamma H^2(\gamma+1)\big\} + \delta^4\big\{Va + H(2\gamma+1)\big\} + \delta^6\big] \tag{3.11}$$

$$A_4 = -a^2\,Le\,Ra_S\,Va\big[\alpha^2\big\{\delta^2(Le-1) + H\,Le\big\}\omega_i^2 + \delta^6(Le-1) + \delta^4 H\big\{Le + 2\gamma(Le-1)\big\} + \delta^2\gamma H^2\big\{\gamma(Le-1) + Le - \alpha\big\}\big] \tag{3.12}$$

Since Ra_T is a physical quantity, it must be real. Hence, from Eq. (3.5) it follows that either $\omega_i = 0$ (steady onset) or $\Delta_2 = 0$ ($\omega_i \neq 0$, oscillatory onset).

3.1. Stationary convection

The direct bifurcation (steady onset) corresponds to $\omega_i = 0$ and the steady convection occurs at

$$Ra_T^{St} = \frac{[\delta^2 + H(\gamma+1)](\delta^4 + a^2\,Le\,Ra_S)}{a^2(\delta^2 + \gamma H)} \tag{3.13}$$

It is worth mentioning that the stationary Rayleigh number is independent of the diffusivity ratio of the fluid and solid phases and also the Vadasz number. When $H \to \infty$, Eq. (3.13) gives

$$Ra_T^{St} = \left(\frac{1+\gamma}{\gamma}\right)\left(\frac{(\pi^2 + a^2)^2}{a^2} + Le\,Ra_S\right)$$

Using the definition (2.23) the above equation takes the form

$$Ra_{T\,\mathrm{LTE}}^{St} = Ra_T^{St}\left(\frac{\gamma}{\gamma+1}\right) = \frac{(\pi^2 + a^2)^2}{a^2} + Le\,Ra_S \tag{3.14}$$

which is a classical result obtained by Nield [2] for the problem of double diffusive convection in a porous layer with LTE model. For $Ra_S = 0$, Eq. (3.14) gives

$$Ra_{T\,\mathrm{LTE}}^{\mathrm{St}} = \frac{(\pi^2 + a^2)^2}{a^2} \tag{3.15}$$

the classical result of Horton and Rogers [43] and Lapwood [44] for single component convection in a porous layer.

If we set $Ra_S = 0$ in Eq. (3.13) we obtain

$$Ra_T^{\mathrm{St}} = \frac{(\pi^2 + a^2)^2}{a^2}\left(\frac{(\pi^2 + a^2) + H(1+\gamma)}{(\pi^2 + a^2 + \gamma H)}\right) \tag{3.16}$$

This is identical with the result of Banu and Rees [29] for the single component convection in a porous layer with the local thermal non-equilibrium model.

The stationary Rayleigh number Ra_T^{St} given by Eq. (3.13) attains the critical value for the wavenumber $a_c = \sqrt{x}$, which satisfies the equation

$$\begin{aligned} &x^4 + 2(\pi^2 + \gamma H)x^3 + H\big[\gamma(1+\gamma)H + \pi^2(2\gamma - 1) \\ &\quad - Le\,Ra_S\big]x^2 - 2\pi^4\big[\pi^2 + H(1+\gamma)\big]x \\ &\quad - \pi^4(\pi^2 + \gamma H)\big[\pi^2 + H(1+\gamma)\big] = 0 \end{aligned} \tag{3.17}$$

Now we discuss the asymptotic analysis for both small and large values of H. The small H physically represents that there is almost no transfer of heat between the fluid and solid phase. The solid phase ceases to affect the thermal field of the fluid, which is free to act independently. On the other hand for large H, the solid phase and the fluid phase have nearly identical temperatures and may be treated as a single phase. The respective mathematical problems are identical except for a rescaling of Ra_T^{St}. The expression for the Rayleigh number and the corresponding wavenumber for small as well as large values of the interphase heat transfer coefficient H are obtained and the same are discussed below.

3.1.1. Case 1: For very small values of H

When H is very small the critical value of the Rayleigh number Ra_T^{St} is slightly above the critical value for the local thermal equilibrium case. Accordingly we expand Ra_T^{St} given by Eq. (3.13) in a power series in H as

$$\begin{aligned} Ra_T^{\mathrm{St}} &= \frac{1}{a^2}\big[(\pi^2 + a^2)^2 + a^2 Le\,Ra_S\big] \\ &\quad \times \left\{1 + \frac{H}{(\pi^2 + a^2)} - \frac{\gamma H^2}{(\pi^2 + a^2)^2} + \cdots\right\} \end{aligned} \tag{3.18}$$

To minimize Ra_T^{St} up to $O(H^2)$ we set $\partial Ra_T^{\mathrm{St}}/\partial a = 0$ and obtain an expression of the form

$$\begin{aligned} &(a^4 - \pi^4) - \left[\pi^2 + \frac{a^4 Le\,Ra_S}{(\pi^2 + a^2)^2}\right]H + \gamma\left[1 + \frac{2a^4 Le\,Ra_S}{(\pi^2 + a^2)^3}\right]H^2 \\ &\quad + \cdots = 0 \end{aligned} \tag{3.19}$$

We also expand a in power series of H as

$$a = a_0 + a_1 H + a_2 H^2 + \cdots \tag{3.20}$$

where $a_0 = \pi$ is critical wavenumber for the local thermal equilibrium case. Substituting Eq. (3.20) into Eq. (3.19), then equating the coefficients of the same powers of H we obtain the values of a_1 and a_2, so that

$$\begin{aligned} a_c &= \pi + H\left[\frac{1}{4\pi} + \frac{Le\,Ra_S}{16\pi^3}\right] - \frac{H^2}{512\pi^7}\big[(4\pi^2 + Le\,Ra_S) \\ &\quad \times \big\{4\pi^2(8\gamma + 3) - Le\,Ra_S\big\}\big] + \cdots \end{aligned} \tag{3.21}$$

Using this in Eq. (3.18) one can obtain the critical Rayleigh number for small H.

3.1.2. Case 2: For very large values of H

For large values of H, expression for the stationary Rayleigh number takes the form

$$\begin{aligned} Ra_T^{\mathrm{St}} &= \frac{1}{a^2}\big[(\pi^2 + a^2)^2 + a^2 Le\,Ra_S\big]\left(\frac{1+\gamma}{\gamma}\right) \\ &\quad \times \left\{1 - \frac{(\pi^2 + a^2)}{\gamma(1+\gamma)}H^{-1} + \frac{(\pi^2 + a^2)^2}{\gamma^2(1+\gamma)}H^{-2} + \cdots\right\} \end{aligned} \tag{3.22}$$

We minimize this with respect to a in a similar way as we did in the small H case

$$\begin{aligned} &\left(\frac{1+\gamma}{\gamma}\right)(a^4 - \pi^4) - \frac{1}{\gamma^2}\bigg[(\pi^2 + a^2)(2a^2 - \pi^2) \\ &\quad + \frac{a^4 Le\,Ra_S}{(\pi^2 + a^2)}\bigg]H^{-1} + \frac{1}{\gamma^3}\big[(\pi^2 + a^2)^2(3a^2 - \pi^2) \\ &\quad + 2a^4 Le\,Ra_S\big]H^{-2} + \cdots = 0 \end{aligned} \tag{3.23}$$

Similarly we expand a in the form

$$a = \pi + \frac{a_1}{H} + \frac{a_2}{H^2} + \cdots \tag{3.24}$$

Substituting Eq. (3.24) into Eq. (3.23) and equating the coefficients of like powers of H^{-1} we find a_1 and a_2 in the form

$$\begin{aligned} a_1 &= \frac{1}{\gamma(1+\gamma)}\left[\pi^3 + \frac{\pi\, Le\,Ra_S}{4}\right] \\ a_2 &= \frac{1}{32\gamma^2(1+\gamma)^2}\big[16\pi^5(1 - 8\gamma) \\ &\quad + \pi\, Le\,Ra_S\big\{8\pi^2(3 - 4\gamma) + 5\,Le\,Ra_S\big\}\big] \end{aligned}$$

Again with these values of a_1 and a_2, we compute the critical wavenumber a_c using Eq. (3.24) and finally using this value of a_c, one can obtain the critical Rayleigh number Ra_T^{St} for stationary convection from Eq. (3.22) for large H. Detailed quantitative comparison of the exact values of stationary critical wavenumber and Rayleigh number computed from Eqs. (3.13) and (3.17) has been made with their asymptotic values for the small and large values of H for different values of γ and Ra_S in Tables 1 and 2. It is important to note that there is an excellent agreement between these two values.

3.2. Oscillatory convection

For oscillatory onset $\Delta_2 = 0$ ($\omega_i \neq 0$) and this gives a dispersion relation of the form (on dropping the subscript i)

$$A(\omega^2)^2 + B(\omega^2) + C = 0 \tag{3.25}$$

Table 1
Comparison of the exact and asymptotic values of critical wavenumber and the critical Rayleigh number for small values of H for the stationary convection ($Le = 2$)

Ra_S	γ	$\log_{10} H$	$a_c^{St}(E)$	$a_c^{St}(A)$	$Ra_{T_c}^{St}(E)$	$Ra_{T_c}^{St}(A)$
10	0.01	−5	3.14159	3.14159	59.4784	59.4784
		−4	3.14160	3.14160	59.4787	59.4787
		−3	3.14171	3.14171	59.4814	59.4814
		−2	3.14279	3.14279	59.5085	59.5085
		−1	3.15354	3.15354	59.7792	59.7792
		0	3.25774	3.25774	62.4355	62.4355
		1	4.06442	4.06438	85.6198	85.9195
	1.0	−5	3.14159	3.14159	59.4784	59.4784
		−4	3.14160	3.14160	59.4787	59.4787
		−3	3.14171	3.14171	59.4814	59.4814
		−2	3.14279	3.14279	59.5085	59.5085
		−1	3.15342	3.15342	59.7777	59.7777
		0	3.24731	3.24652	62.3012	62.2945
		1	3.61046	3.12330	78.5567	74.3443
100	0.01	−5	3.14160	3.14160	239.479	239.479
		−4	3.14164	3.14164	239.480	239.480
		−3	3.14208	3.14208	239.491	239.491
		−2	3.14642	3.14642	239.600	239.600
		−1	3.18997	3.18997	240.682	240.682
		0	3.62351	3.62351	250.731	250.731
		1	6.17243	6.17240	314.566	314.566
	1.0	−5	3.14160	3.14160	239.479	239.479
		−4	3.14164	3.14164	239.480	239.480
		−3	3.14208	3.14208	239.491	239.491
		−2	3.14642	3.14642	239.600	239.600
		−1	3.18950	3.18949	240.676	240.676
		0	3.58416	3.58146	250.295	250.275
		1	5.36582	2.10409	303.683	297.918

E indicates exact and A, asymptotic.

Table 2
Comparison of the exact and asymptotic values of critical wavenumber and the critical Rayleigh number for large values of H for stationary convection ($Le = 2$)

Ra_S	γ	$\log_{10} H$	$a_c^{St}(E)$	$a_c^{St}(A)$	$Ra_{T_c}^{St}(E)$	$Ra_{T_c}^{St}(A)$
10	0.01	5	3.18864	3.18858	5891.35	5891.40
		7	3.14206	3.14206	6006.15	6006.15
		9	3.14160	3.1416	6007.31	6007.31
		11	3.14159	3.14159	6007.32	6007.32
		13	3.14159	3.14159	6007.32	6007.32
	1.0	3	3.16469	3.16466	117.801	117.802
		5	3.14183	3.14183	118.945	118.945
		7	3.14159	3.14159	118.957	118.957
		9	3.14159	3.14159	118.957	118.957
50	0.01	5	3.35625	3.35591	23708.8	23709.0
		7	3.14346	3.14346	24182.6	24182.6
		9	3.14161	3.14161	24187.3	24187.3
		11	3.14159	3.14159	24187.3	24187.3
		13	3.14159	3.14159	24187.3	24187.3
	1.0	3	3.24011	3.23997	474.251	474.253
		5	3.14253	3.14253	478.910	478.910
		7	3.14160	3.14160	478.956	478.956
		9	3.14159	3.14159	478.957	478.957
		11	3.14159	3.14159	478.957	478.957

E indicates exact and A, asymptotic.

where

$$
\begin{aligned}
A &= \alpha^2 Le^2\big(\delta^2 + H + Va\big) \\
B &= \delta^6(\alpha^2 + Le^2) + \delta^4\big\{(H + Va)(\alpha^2 + Le^2) + 2\,Le^2\,\gamma H\big\} \\
&\quad + \delta^2\,Le^2\,\gamma H\big\{2Va + H(1+\gamma)\big\} \\
&\quad + a^2\alpha^2\,Le\,Ra_S\,Va\big\{\delta^2(1 - Le) + Le\,H\big\} \\
&\quad + \gamma H^2\,Le^2\,Va(\alpha + \gamma) \\
C &= \delta^4\big[\delta^6 + \delta^4\big\{Va + H(1+2\gamma)\big\} + \delta^2\big\{2\,Va + H(1+\gamma)\big\} \\
&\quad + \gamma H^2\,Va(\alpha + \gamma)\big] + a^2\,Le\,Ra_S\,Va\big[\delta^4(1 - Le) \\
&\quad + \delta^2 H\big\{2\gamma(1 - Le) - Le\big\} + \gamma H^2\big\{(\alpha - Le) \\
&\quad + \gamma(1 - Le)\big\}\big]
\end{aligned}
$$

Now Eq. (3.5) with $\Delta_2 = 0$, gives

$$Ra_T^{Osc} = A_0(A_1 + A_2) \tag{3.26}$$

with the values of A_0, A_1 and A_2 given by Eqs. (3.8)–(3.10). For the oscillatory convection to occur ω^2 must be positive. Since Eq. (3.25) is a quadratic in ϖ^2, it can give rise to more than one positive root, for fixed values of the governing parameters. This has important implications for the linear stability of double-diffusive porous layer. Thus in case this equation has two real positive roots then there exist two oscillatory neutral solutions. From Descartes' rule of signs in order for (3.25) to have two positive real roots, it is necessary that $B < 0$ and $C > 0$. We find the oscillatory neutral solutions from Eq. (3.26). It proceeds as follows: First determine the number of positive solutions of Eq. (3.25). If there are none, then no oscillatory instability is possible. If there are two, then the minimum (over a^2) of Eq. (3.26) with ω^2 given by Eq. (3.25) gives the oscillatory neutral Rayleigh number $Ra_{T,c}^{Osc}$ corresponding to the critical wavenumber a_c and the critical frequency of the oscillation ω_c^2. The analytical expression for oscillatory Rayleigh number given by Eq. (3.26) is evaluated at $a = a_c$ and $\omega^2 = \omega_c^2$ for various values of the physical parameters in order to know their effects on the onset of oscillatory convection.

4. Finite amplitude steady convection

In this section we consider the nonlinear analysis using a truncated representation of Fourier series considering only two terms. Although the linear stability analysis is sufficient for obtaining the stability condition of the motionless solution and the corresponding eigenfunctions describing qualitatively the convective flow, it cannot provide information about the values of the convection amplitudes, nor regarding the rate of heat transfer. To obtain this additional information, we perform the nonlinear analysis, which is useful to understand the physical mechanism with minimum amount of mathematical analysis and is a step forward towards understanding full nonlinear problem. There is a large body of literature available on the finite amplitude thermal convection in porous medium with local thermal equilibrium condition for both single and two component systems (see e.g. [3,38,40–42] and [45]). The method proposed by these authors has been adopted here in this paper to study the effect of local thermal non-equilibrium on double diffusive convection in a porous layer.

A minimal double Fourier series which describes the finite amplitude steady-state convection is given by

$$\psi = A\sin(ax)\sin(\pi z) \tag{4.1}$$

$$T_f = B_1\cos(ax)\sin(\pi z) + B_2\sin(2\pi z) \tag{4.2}$$

$$T_s = B_3\cos(ax)\sin(\pi z) + B_4\sin(2\pi z) \tag{4.3}$$

$$S = B_5\cos(ax)\sin(\pi z) + B_6\sin(2\pi z) \tag{4.4}$$

where the steady-state amplitudes A, B_i's are constants and are to be determined from the dynamics of the system. Substituting Eqs. (4.1)–(4.4) into the steady part of coupled nonlinear system of partial differential equations (2.19)–(2.22) and equating the coefficients of like terms we obtain the following nonlinear system equations

$$(\pi^2 + a^2)A + a\,Ra_T\,B_1 - a\,Ra_S\,B_5 = 0 \tag{4.5}$$

$$aA + \left[(\pi^2 + a^2) + H\right]B_1 - HB_3 + \pi aAB_2 = 0 \tag{4.6}$$

$$2[4\pi^2 + H]B_2 - 2HB_4 - \pi aAB_1 = 0 \tag{4.7}$$

$$\gamma HB_1 - \left[(\pi^2 + a^2) + \gamma H\right]B_3 = 0 \tag{4.8}$$

$$\gamma HB_2 - [4\pi^2 + \gamma H]B_4 = 0 \tag{4.9}$$

$$aA + \frac{1}{Le}(\pi^2 + a^2)B_5 + \pi aAB_6 = 0 \tag{4.10}$$

$$\frac{8\pi^2}{Le}B_6 - \pi aAB_5 = 0 \tag{4.11}$$

The steady state solutions are useful because they predict that a finite amplitude solution to the system is possible for subcritical values of the Rayleigh number and that the minimum values of Ra_T for which a steady solution is possible lies below the critical values for instability to either a marginal state or an overstable infinitesimal perturbation. Elimination of all amplitudes, except A, yields

$$A\left\{(\pi^2 + a^2) - a^2\,Ra_T\left[\frac{(\pi^2 + a^2)\{\pi^2 + a^2 + H(1+\gamma)\}}{(\pi^2 + a^2 + \gamma H)} + \frac{a^2(4\pi^2 + \gamma H)}{4\pi^2 + H(1+\gamma)}\left(\frac{A^2}{8}\right)\right]^{-1} + a^2\,Ra_S\left[\frac{(\pi^2 + a^2)}{Le} + a^2\,Le\left(\frac{A^2}{8}\right)\right]^{-1}\right\} = 0 \tag{4.12}$$

The solution $A = 0$ corresponds to pure conduction, which we know to be a possible solution though it is unstable when Ra_T is sufficiently large. The remaining solutions are given by

$$\frac{A^2}{8} = \frac{1}{2x_1}\left[-x_2 + \sqrt{x_2^2 - 4x_1x_3}\,\right] \tag{4.13}$$

where

$$x_1 = \frac{a^4\delta^2\,Le(4\pi^2 + \gamma H)}{4\pi^2 + H(\gamma + 1)}$$

$$x_2 = \frac{a^2\delta^4\,Le[\delta^2 + H(\gamma + 1)]}{\delta^2 + \gamma H} + \frac{a^2\delta^4(4\pi^2 + \gamma H)}{Le[4\pi^2 + H(\gamma + 1)]} + \frac{a^4\,Ra_S(4\pi^2 + \gamma H)}{[4\pi^2 + H(\gamma + 1)]} - a^4\,Le\,Ra_T$$

$$x_3 = \left(\frac{\delta^6}{Le} + a^2\delta^2\,Ra_S\right)\left(\frac{\delta^2 + H(\gamma + 1)}{\delta^2 + \gamma H}\right) - \frac{a^2\delta^2}{Le}\,Ra_T$$

When we let the radical in the above equation to vanish, we obtain an expression for finite amplitude Rayleigh number Ra_T^F, which characterizes the onset of finite amplitude steady motions. The finite amplitude Rayleigh number can be obtained in the form

$$Ra_T^F = \frac{1}{2y_1}\left[-y_2 + \sqrt{y_2^2 - 4y_1y_3}\,\right] \tag{4.14}$$

where

$$y_1 = a^4\,Le^2(\delta^2 + \gamma H)\left[4\pi^2 + H(\gamma + 1)\right]$$

$$y_2 = -2a^2\left[a^2\,Le\,Ra_S(\delta^2 + \gamma H)(4\pi^2 + \gamma H) + \delta^4\left\{H^2\left[Le(\gamma + 1) + Le\right]\left[Le(\gamma + 1) - \gamma\right] + 4\pi^2\delta^2(Le^2 - 1) + H(\delta^2 + 4\pi^2)\left[Le^2(\gamma + 1) - \gamma\right]\right\}\right]$$

$$y_3 = \frac{1}{Le^2(\delta^2 + \gamma H)[4\pi^2 + H(\gamma + 1)]}\left[a^2\,Le\,Ra_S(4\pi^2 + \gamma H) \times (\delta^2 + \gamma H) - \delta^4\left\{H^2\left[Le(\gamma + 1) + Le\right]\left[Le(\gamma + 1) - \gamma\right] + 4\pi^2\delta^2(Le^2 - 1) - H(\delta^2 + 4\pi^2)\left[Le^2(\gamma + 1) - \gamma\right]\right\}\right]^2$$

5. Heat and mass transports

Once we know the amplitude we can find the heat and mass transfer. In the study of convection in porous medium, the quantification of heat and mass transports is important. This is because onset of convection, as Rayleigh number is increased, is more readily detected by its effect on the heat transport. In the basic state, heat and mass transport is by conduction alone.

If $\bar{H}$ and $\bar{J}$ are the rate of heat transport per unit area for the fluid phase and mass transport per unit area respectively, then

$$\bar{H} = -\kappa_f\left\langle\frac{\partial T_{f\,\mathrm{total}}}{\partial z}\right\rangle_{z=0} \tag{5.1}$$

$$\bar{J} = -D\left\langle\frac{\partial S_{\mathrm{total}}}{\partial z}\right\rangle_{z=0} \tag{5.2}$$

where the angular bracket corresponds to a horizontal average and

$$T_{f\,\mathrm{total}} = T_l - \Delta T\frac{z}{d} + T_f(x, z) \tag{5.3}$$

$$S_{\mathrm{total}} = S_l - \Delta S\frac{z}{d} + S(x, z) \tag{5.4}$$

Substituting Eqs. (4.2) and (4.4) into Eqs. (5.3) and (5.4) and using the resultant equations in Eqs. (5.1) and (5.2), we get

$$\bar{H} = \frac{\kappa_f\,\Delta T}{d}(1 - 2\pi B_2) \tag{5.5}$$

$$\bar{J} = \frac{D\Delta S}{d}(1 - 2\pi B_6) \tag{5.6}$$

The Nusselt number for the fluid phase and Sherwood number are defined by

$$Nu = \frac{\bar{H}}{\kappa_f \Delta T/d} = 1 - 2\pi B_2 \tag{5.7}$$

$$Sh = \frac{\bar{J}}{D\Delta S/d} = 1 - 2\pi B_6 \tag{5.8}$$

Writing B_2 and B_6 in terms of A, using Eqs. (4.5)–(4.11) and substituting into Eqs. (5.7) and (5.8), we obtain

$$\begin{aligned} Nu = 1 + 2a^2(A^2/8)\big[(\delta^2+\gamma H)(4\pi^2+\gamma H) \\ /\big\{\delta^2\big[\delta^2+H(\gamma+1)\big]\big[4\pi^2+H(1+\gamma)\big] \\ + a^2\big(\delta^2+\gamma H\big)(4\pi^2+\gamma H)(A^2/8)\big\}\big] \end{aligned} \tag{5.9}$$

$$Sh = 1 + 2a^2(A^2/8)\left[\frac{Le^2}{\delta^2 + a^2 Le^2(A^2/8)}\right] \tag{5.10}$$

The second term on the right-hand side of Eqs. (5.9) and (5.10) represent the convective contribution to heat and mass transport respectively. It is obvious that $Nu = Sh = 1$ for all $Ra_T \leqslant Ra^{\mathrm{St}}_{T,c}$, indicating that the convection heat and mass transfer branches off from the conductive heat transfer line at the critical value of the thermal Rayleigh number. It is important to note that our finite amplitude analysis is valid for thermal Rayleigh number around the convection threshold. Therefore the Nusselt number and the Sherwood number in the present study are limited by an upper bound value of 3. Better results can only be obtained by including more number of terms in the Fourier series representation, which allows the variation of wave number as the value of thermal Rayleigh number varies.

Now we intend to obtain the quantification of heat and mass transports for the limiting cases $H \to 0$ and $H \to \infty$.

When $H \to 0$, Eq. (5.9) yields

$$Nu = 1 + 2a^2(A^2/8)\left[\frac{1}{a^2 + \delta^2(A^2/8)}\right] \tag{5.11}$$

while Eq. (5.10) remains unchanged, with $(A^2/8)$ given by Eq. (4.13) where

$$x_1 = a^4\delta^2 Le$$

$$x_2 = \frac{a^2}{Le}\big[\delta^4(1+Le^2) + a^2 Le\,\gamma(Ra_S - Ra_T\,Le)\big]$$

$$x_3 = \frac{\delta^2}{Le}\big[\delta^4 + a^2(Le\,Ra_S - Ra_T)\big].$$

Further, when $H \to \infty$, Eqs. (5.9) reads

$$Nu = 1 + 2a^2(A^2/8)\left[\frac{1}{a^2 + \delta^2(1+1/\gamma)^2(A^2/8)}\right] \tag{5.12}$$

and Eq. (5.10) remains invariant, with $(A^2/8)$ given by Eq. (4.13) where

$$x_1 = \left(\frac{\gamma}{\gamma+1}\right)a^4\delta^2 Le$$

$$\begin{aligned} x_2 = \frac{a^2}{Le\,\gamma(\gamma+1)}\big[\delta^4\big\{\gamma^2 + Le^2(\gamma+1)^2\big\} \\ + a^2 Le\,\gamma\big\{\gamma\,Ra_S - Ra_T\,Le(\gamma+1)\big\}\big] \end{aligned}$$

$$x_3 = \frac{\delta^2}{\gamma\,Le}\big[\delta^4(\gamma+1) + a^2\big\{Le\,Ra_S(\gamma+1) - \gamma\,Ra_T\big\}\big]$$

Detailed quantitative comparison of the exact values of Nusselt number and Sherwood number computed from the above equations for the cases of $H \to 0$ and $H \to \infty$ has been made with

Table 3
Comparison of the exact and asymptotic values of Nusselt and Sherwood number for small values of H for different values of Ra_S and γ ($Le = 2$, $Ra_T = 2 \times Ra^{\mathrm{St}}_{T_c}$)

Ra_S	γ	$\log_{10} H$	$Nu(E)$	$Nu(A)$	$Sh(E)$	$Sh(A)$
10	0.01	−4	2.29606	2.29606	2.76090	2.76090
		−3	2.29609	2.29606	2.76091	2.76091
		−2	2.29644	2.29609	2.76107	2.76107
		−1	2.29988	2.29634	2.76266	2.76261
		0	2.33111	2.29847	2.77679	2.77683
		1	2.48920	2.29841	2.84204	2.85480
	1.0	−4	2.29606	2.29606	2.76090	2.76090
		−3	2.29609	2.29606	2.76091	2.76091
		−2	2.29644	2.29609	2.76107	2.76107
		−1	2.29986	2.29634	2.76265	2.76261
		0	2.32971	2.29835	2.77616	2.77626
		1	2.46644	2.25957	2.83324	2.83251
50	0.01	−4	2.68494	2.68494	2.91068	2.91068
		−3	2.68496	2.68494	2.91069	2.91069
		−2	2.68512	2.68496	2.91074	2.91075
		−1	2.68670	2.68518	2.91125	2.91140
		0	2.69912	2.68571	2.91521	2.91673
		1	2.71565	2.64324	2.92043	2.93405
	1.0	−4	2.68494	2.68494	2.91068	2.91068
		−3	2.68496	2.68494	2.91069	2.91069
		−2	2.68512	2.68496	2.91074	2.91075
		−1	2.68669	2.68518	2.91124	2.91140
		0	2.69881	2.68588	2.91511	2.91660
		1	2.73525	2.67266	2.92652	2.94121

E indicates exact and A, asymptotic.

Table 4
Comparison of the exact and asymptotic values of Nusselt and Sherwood number for large values of H for different values of Ra_S and γ ($Le = 2$, $Ra_T = 2 \times Ra^{\mathrm{St}}_{T_c}$)

R_S	γ	$\log_{10} H$	$Nu(E)$	$Nu(A)$	$Sh(E)$	$Sh(A)$
10	0.01	4	2.17399	2.34023	2.99997	2.99997
		6	2.33495	2.33625	2.99998	2.99998
		8	2.33624	2.33625	2.99998	2.99998
		10	2.33625	2.33625	2.99998	2.99998
		12	2.33625	2.33625	2.99998	2.99998
	1.0	4	2.32522	2.32587	2.93832	2.93823
		6	2.32589	2.32590	2.93841	2.93840
		8	2.32590	2.32590	2.93841	2.93841
		10	2.32590	2.32590	2.93841	2.93841
		12	2.32590	2.32590	2.93841	2.93841
50	0.01	5	2.71094	2.71669	2.99999	2.99999
		6	2.71640	2.71695	2.99999	2.99999
		7	2.71690	2.71695	2.99999	2.99999
		8	2.71695	2.71695	2.99999	2.99999
		9	2.71695	2.71695	2.99999	2.99999
	1.0	5	2.70637	2.70913	2.97872	2.97834
		6	2.70912	2.70940	2.97895	2.97891
		7	2.70939	2.70942	2.97897	2.97897
		8	2.70942	2.70942	2.97897	2.97897
		9	2.70942	2.70942	2.97897	2.97897

E indicates exact and A, asymptotic.

their asymptotic values computed from Eqs. (5.9) and (5.10) by substituting the critical values of thermal Rayleigh number and wavenumber for the small and large values of H respectively for different values of γ and Ra_S, in Tables 3 and 4. It is important to note that there is an excellent agreement between these two values.

6. Results and discussion

An analytical study of linear and nonlinear double diffusive convection in a horizontal fluid-saturated porous layer is carried out by considering a thermal non-equilibrium model. The onset criterion for both marginal and oscillatory convection is derived in the linear theory. In the realm of nonlinear theory the finite amplitude steady convection is discussed. The expression for finite amplitude Rayleigh number is derived analytically. The effect of both solute diffusion and thermal non-equilibrium on the stability of the system is investigated. It is found that in most of the situations the instability sets in via finite amplitude motions, prior to the marginal or oscillatory convection. However, for large values of solute Rayleigh number and Lewis number the finite amplitude motions become weaker and the onset of double diffusive convection ceases to be in the form of finite amplitude convection and the instability sets in via oscillatory mode. Further the measure of heat and mass transport across the layer is determined by evaluating the Nusselt number and the Sherwood number for the case of steady convection.

The neutral stability curves in Ra_T–a plane for various parameter values are shown through Figs. 1–6. From these figures it is clear that the neutral curves are connected in a topological sense. This connectedness allows the linear stability criteria to be expressed in terms of the critical Rayleigh number, below which the system is stable and unstable above. The points where the overstable solutions branch off from the stationary convection can be easily identified from these figures. We also observe that for smaller values of the wavenumber each curve is a margin of the oscillatory instability and at some fixed wavenumber depending on the other parameters the overstability disappears and the curve forms the margin of stationary convection.

The effect of inter-phase heat transfer coefficient H on the neutral stability curves is shown in Fig. 1 for the fixed values of solute Rayleigh number, porosity modified conductivity ratio, Lewis number, Vadasz number and ratio of diffusivities. It follows from this figure that for the set of values chosen for the parameters, the onset of convection is through oscillatory state. Further, we observe that the minimum of Rayleigh number for oscillatory mode increases with H, indicating that the effect of inter-phase heat transfer coefficient is to stabilize the system.

In Fig. 2 the effect of solute Rayleigh number Ra_S on the marginal stability curves for the fixed values of other governing parameters is depicted. We observe that for small values of Ra_S the convection sets in first as stationary mode. It is interesting to note that there is a critical value $Ra_S = Ra_S^*$

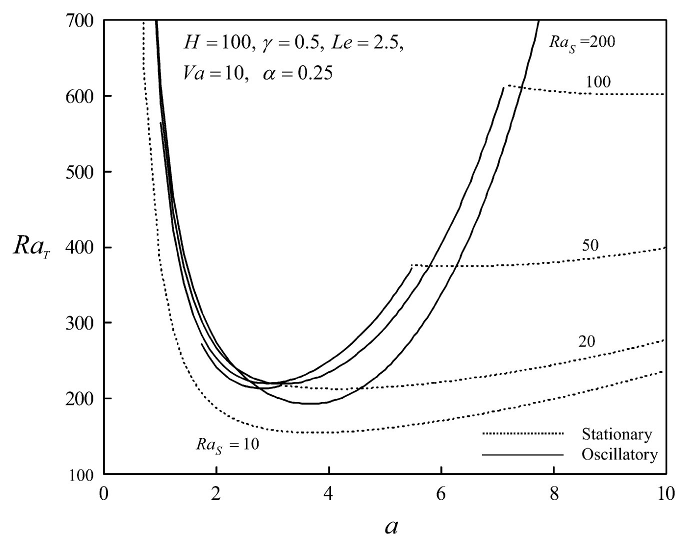

Fig. 2. Neutral stability curves for different values of solute Rayleigh number Ra_S.

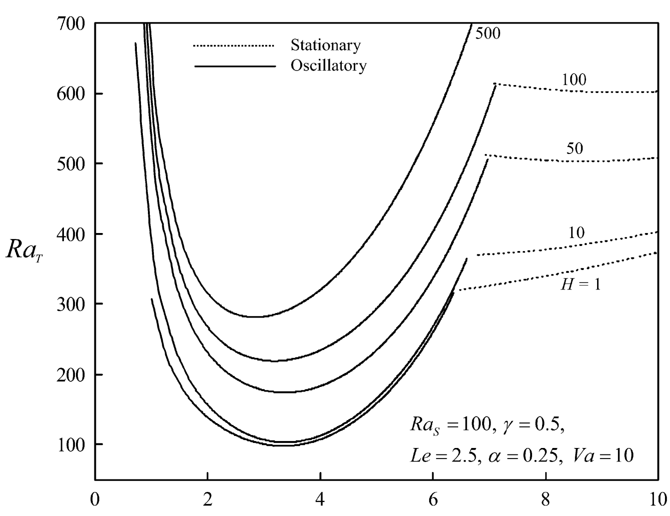

Fig. 1. Neutral stability curves for different values of inter-phase heat transfer coefficient H.

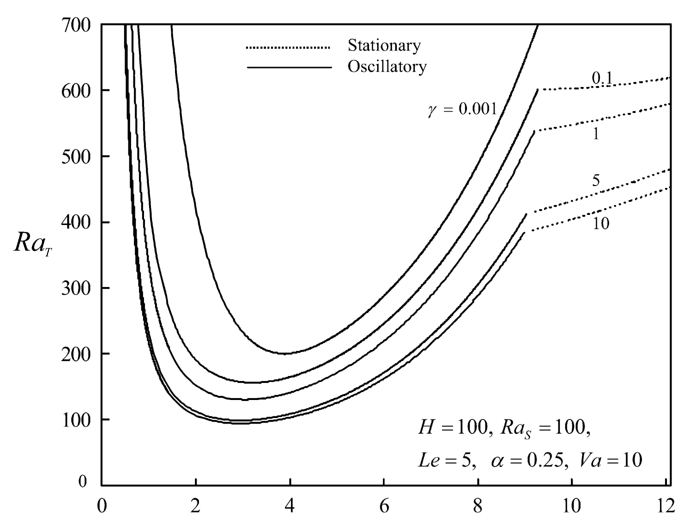

Fig. 3. Neutral stability curves for different values of porosity modified conductivity ratio γ.

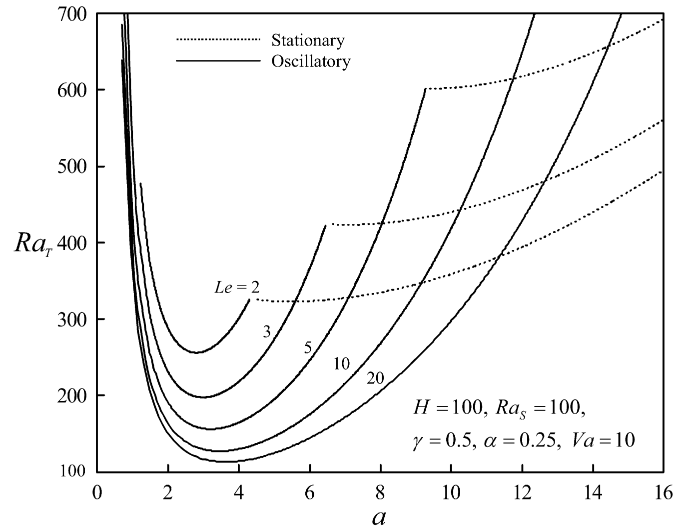

Fig. 4. Neutral stability curves for different values Lewis number *Le*.

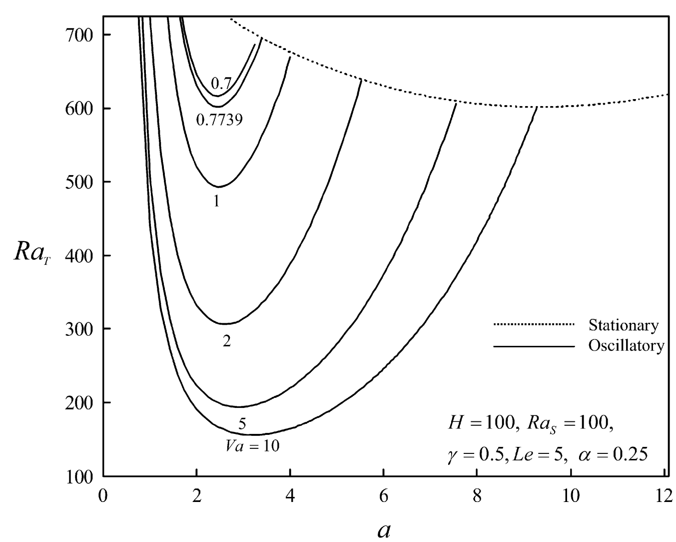

Fig. 5. Neutral stability curves for different values of Vadasz number *Va*.

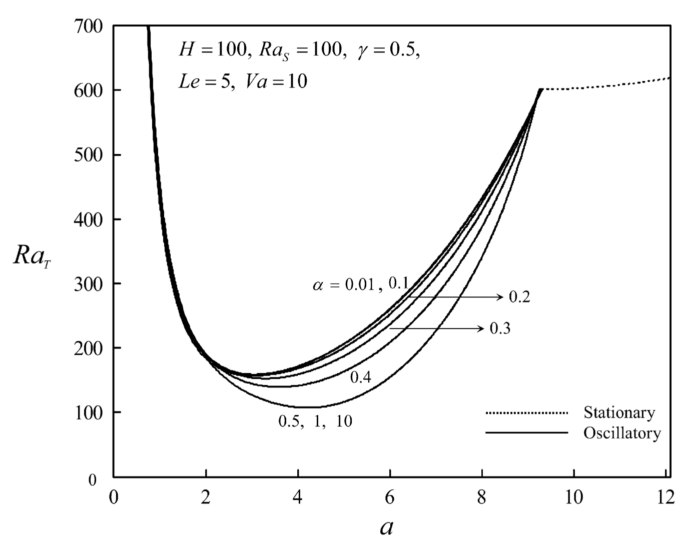

Fig. 6. Neutral stability curves for different values of ratio of diffusivities α.

(e.g., $Ra_S^* = 19.579$ for the fixed values of $H = 100$, $\gamma = 0.5$, $Va = 10$ and $\alpha = 0.25$) such that for $Ra_S < Ra_S^*$ the instability manifests as stationary convection and for $Ra_S \geqslant Ra_S^*$, the onset of instability manifests as oscillatory convection. Therefore, the effect of solute Rayleigh number is to allow the onset of oscillatory convection instead of stationary convection. It is also observed from this figure that the minimum of Rayleigh number for stationary state increases with the solute Rayleigh number. However, in the case oscillatory states it is interesting to note that for moderate values of Ra_S, $Ra_{T,c}$ increases with Ra_S but reverses the trend for fairly higher values of Ra_S.

Fig. 3 indicates the effect of porosity modified conductivity ratio γ on the marginal stability curves. We observe that with the increase in the value of γ the minimum of oscillatory Rayleigh number decreases. The effect of γ therefore, is to advance the onset of oscillatory convection. It is interesting to note that the bifurcation from oscillatory to stationary mode occurs at same value of the wavenumber for all values of γ. The effect of Lewis number on the neutral curves is unveiled in Fig. 4. It is found that the minimum of oscillatory Rayleigh number decreases with *Le*, indicating that the effect of Lewis number is to destabilize the system. Further, with the increasing *Le*, the point where the overstable solutions bifurcate into the stationary motions are shifted towards the higher value, this suggests that the *Le* blows up the region of oscillatory convection. The similar effect is observed with solute Rayleigh number.

The variation of marginal curves for different values of Vadasz number, *Va*, with all other parameters kept fixed, is depicted in Fig. 5. It is important to note that there is a critical value $Va = Va^*$ (e.g., $Va^* = 0.7739$ for the fixed values of $H = 100$, $\gamma = 0.5, Ra_S = 100$ and $\alpha = 0.25$) such that for $Va < Va^*$ the instability manifests as stationary convection and for $Va \geqslant Va^*$, the onset of instability manifests as oscillatory convection. It is clear that the critical value of oscillatory Rayleigh number decreases with the increase in Vadasz number, indicating that, the Vadasz number augments the onset of oscillatory convection. It is also important to note that similar to the R_S and *Le*, the Vadasz number enlarges the region oscillatory convection. In Fig. 6 we display the effect of ratio of diffusivities, α on the neutral stability curves. In this case also the effect similar to that of γ, *Le*, and *Va* is observed. That is, for the set of values chosen for the parameters, the diffusivity ratio enhances the onset of double diffusive convection in the oscillatory region.

The behavior of the critical values of Rayleigh number for stationary, oscillatory and finite amplitude convection and also the wavenumber and frequency of the oscillatory mode is depicted through Figs. 7–9, as the functions of inter-phase heat transfer coefficient H. In general, it is observed that for very small and large values of H the stability criterion is found to be independent of H. However, the effect of H on the stability of the system is significant only for intermediate values of H. The physical reason for this is that when $H \to 0$ there is almost no transfer of heat between the fluid and solid phases and the properties of solid phase have no significant influence on the onset criterion. When $H \to \infty$ the fluid and solid phase have almost equal temperatures and therefore may be treated a sin-

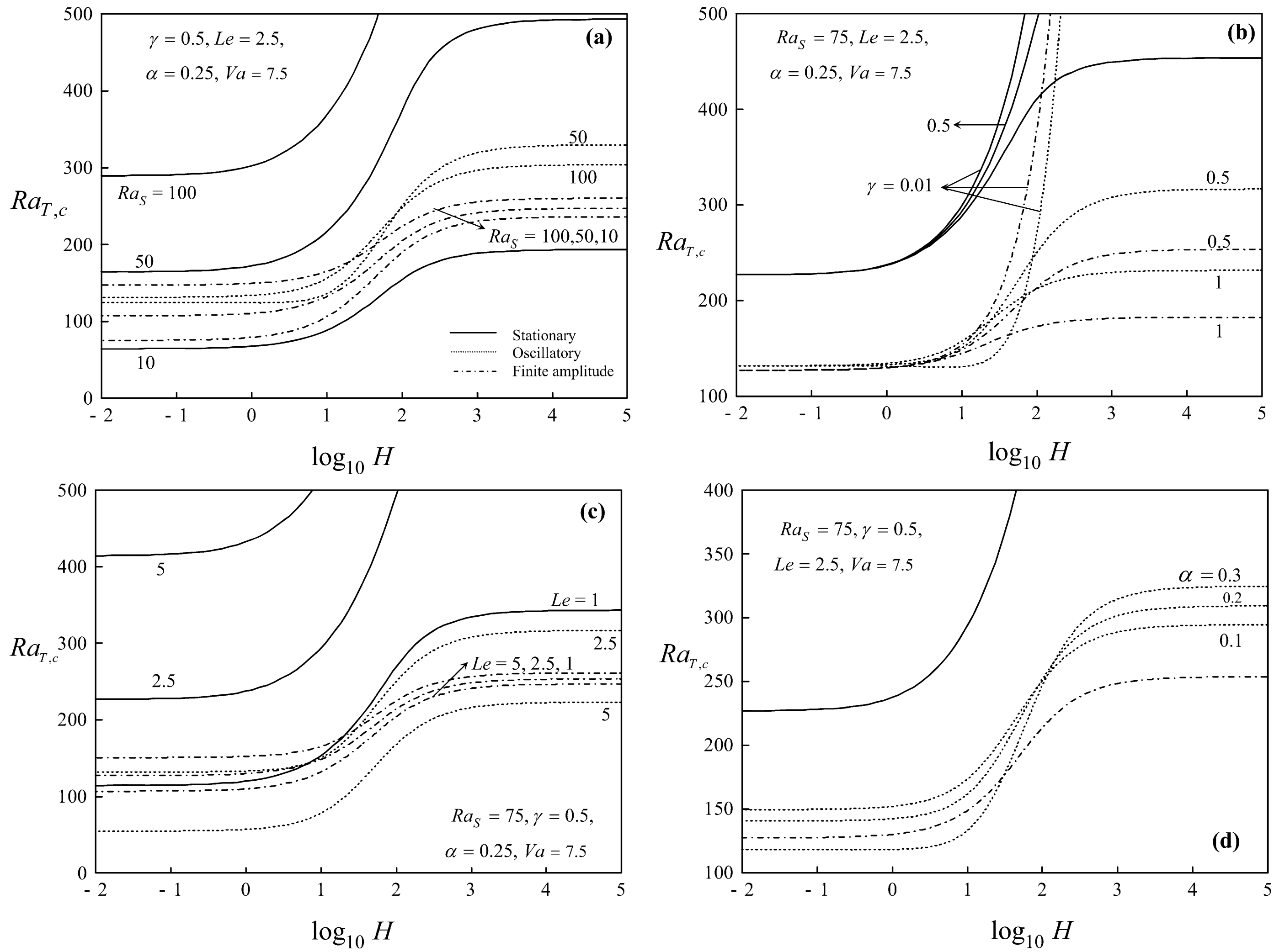

Fig. 7. Variation of critical Rayleigh number $Ra_{T,c}$ with H for different values of (a) Ra_S (b) γ, (c) Le and (d) α.

gle phase. Between these two extremes H gives rise to a strong non-equilibrium effect.

The variation of critical Rayleigh number with inter-phase heat transfer coefficient H for different parameter values is shown in Figs. 7(a)–(d). These figures indicate that the critical Rayleigh number increases from the LTE value when H is small to a LTNE value when H is large. Thus, the inter-phase heat transfer coefficient makes the system more stable for its intermediate values. Fig. 7(a) indicates the effect of Ra_S on the critical Rayleigh number. For small values of solute Rayleigh number ($Ra_S \leqslant 10$) the stationary onset occurs. As the value of Ra_S is increased further the finite amplitude motions become significant and therefore the convection occurs through the finite amplitude motions. For small and moderate values of H, the fairly large Ra_S ($\geqslant 100$) dampens the finite amplitude motions and the overstable mode becomes the most dangerous mode in such case. However for large H (>100) once again the instability sets in through finite amplitude motions. The critical Rayleigh number for both stationary and finite amplitude convection is found to increase with the solute Rayleigh number, indicating that the presence of additional diffusing component stabilizes the system towards the marginal and finite amplitude convection. A similar stabilizing effect of Ra_S is observed in the oscillatory mode only for small and moderate values of H, however a reverse trend is reported for large values of H.

The variation of oscillatory critical Rayleigh number with H for different values of porosity modified conductivity ratio γ is depicted in Fig. 7(b). We observe from this figure that for very small values of H, $Ra_{T,c}$ is independent of γ and is close to that of the LTE case, since for very small values of H, there is no significant transfer of heat between the phases and the onset criterion is not affected by the properties of the solid phase. On the other hand, for large values of H, though the stability criterion is independent of H, the condition for the onset of convection is based on the mean properties of the medium and therefore, the critical Rayleigh number is function of γ. It is observed that for small γ ($\leqslant 0.01$) the instability sets in via finite amplitude motions when H is small, whereas for large values of H the oscillatory onset occurs. For higher values of γ the onset of double diffusive convection is merely through overstable mode. This figure also indicates that for moderate and large values of H, critical Rayleigh number for each of sta-

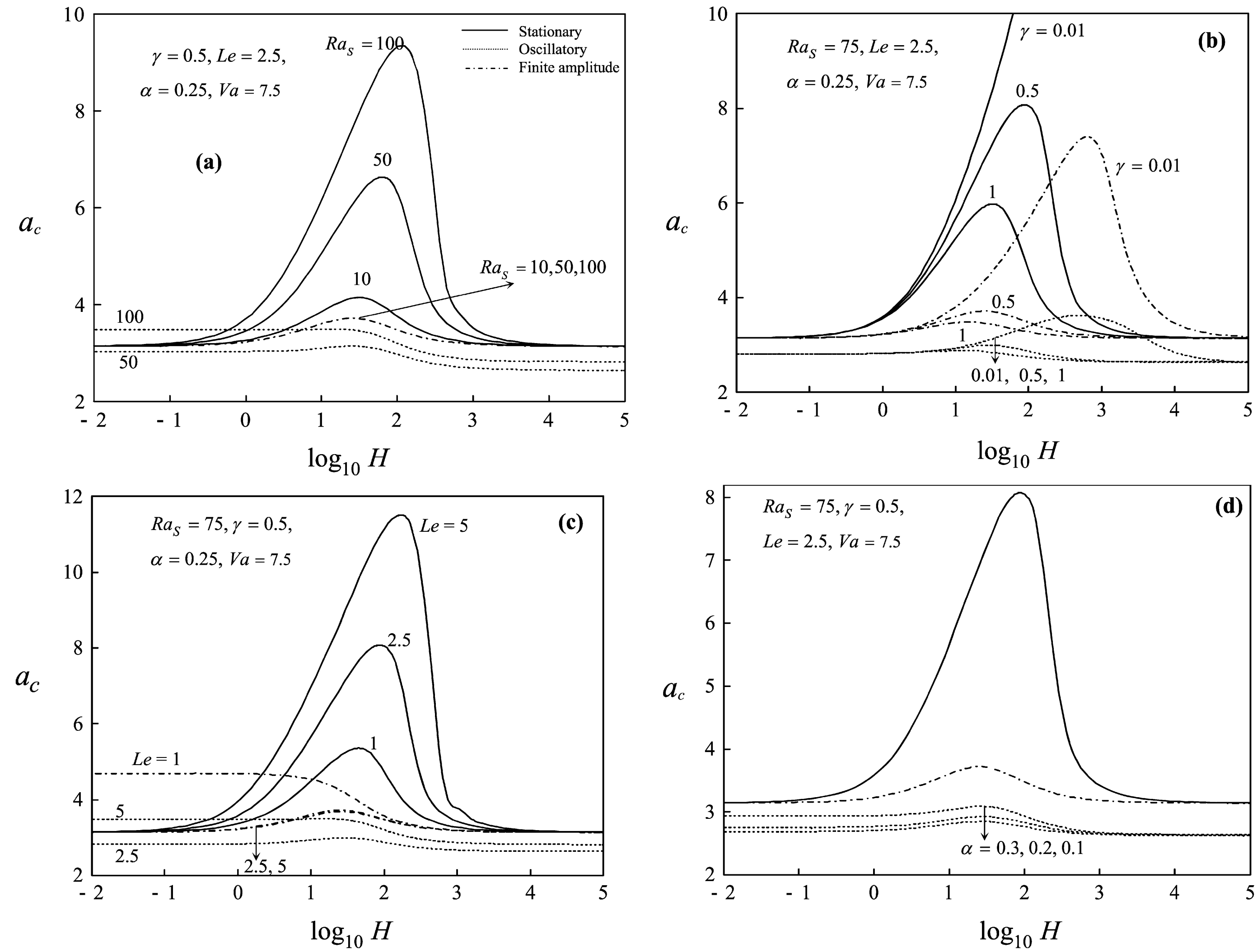

Fig. 8. Variation of critical wavenumber a_c with H for different values of (a) Ra_S, (b) γ, (c) Le and (d) α.

tionary, oscillatory and finite amplitude convection decreases with the increasing values of γ. Therefore the effect of porosity modified conductivity ratio is to reduce the stabilizing effect of inter-phase heat transfer coefficient. It is important to note that for sufficiently large values of γ, the critical Rayleigh number becomes independent of H.

Fig. 7(c) displays the variation of critical Rayleigh number with H for different values of Lewis number. We observe that for small values of Le ($\leqslant 1$) the oscillatory motions are impossible and the stationary onset occurs when H is small while the finite amplitude convection occurs for large H. However, for large values of Le the oscillatory convection is possible prior to both stationary and finite amplitude motion. This figure also reveals that with the increasing values of Le, the critical Rayleigh number for stationary mode increases while that for oscillatory and finite amplitude convection decrease. Therefore, the Lewis number enhances the stability of the double diffusive system in stationary mode while support the oscillatory and finite amplitude convection.

In Fig. 7(d) the variation of $Ra_{T,c}$ with H for different values of diffusivity ratio α is indicated. We observe that for small values of α the onset of convection is through finite amplitude motions whereas for $\alpha \geqslant 0.3$ the oscillatory onset occurs when H is small and for large H once again the finite amplitude motions become significant. It is also observed that the onset of stationary and finite amplitude convection is independent of α. This figure indicates that for moderate and large values of H the oscillatory critical Rayleigh number increases with increasing α while a reverse effect is observed for small values of H. As α increases, the contribution of heat conduction from the solid phase becomes negligible, and therefore the critical Rayleigh number for oscillatory mode increases towards a constant value. The diffusivity ratio therefore reinforces the stabilizing effect of inter-phase heat transfer coefficient in case of the overstable mode.

The variation of critical wavenumber with inter-phase heat transfer coefficient H is shown in Figs. 8(a)–(d) for different parameter values. We observe from these figures that the oscillatory critical wavenumber decreases monotonically from the LTE value when H is small to a LTNE value when H is large, while the critical wavenumber for stationary and finite amplitude mode increases with H to its maximum value and

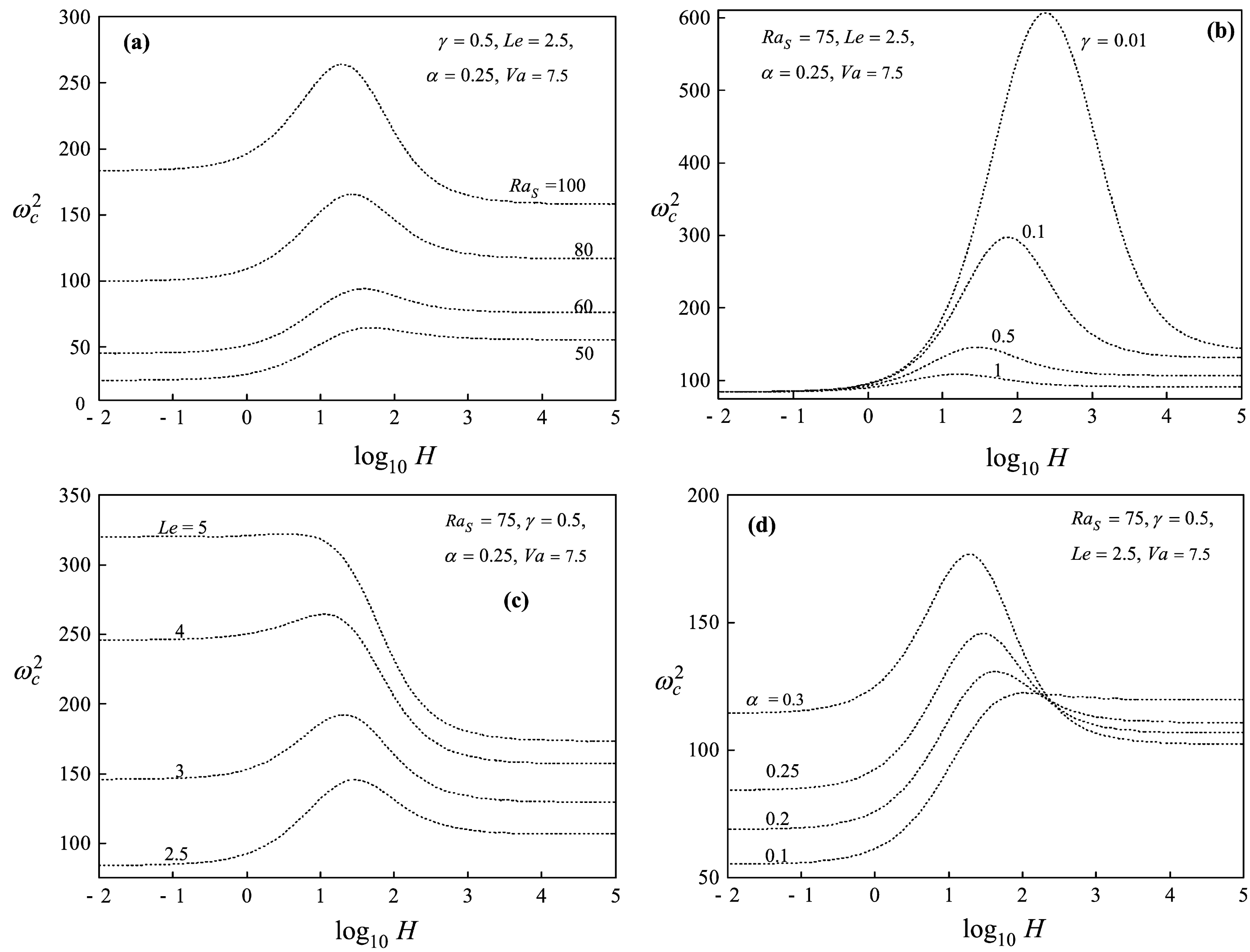

Fig. 9. Variation of critical frequency ω_c^2 with H for different values of (a) Ra_S, (b) γ, (c) Le and (d) α.

then decreases back with further increase in H. The effect of solute Rayleigh number on critical wavenumber is displayed in Fig. 8(a). This figure indicates that the critical wavenumber increases with the increasing Ra_S in both stationary and overstable modes, whereas the effect of Ra_S on critical wavenumber for finite amplitude mode is not significant. We found that the critical wavenumber for the stationary mode approaches to that of LTE case when $H \to 0$ and $H \to \infty$. This is quite obvious as the corresponding physical problems are equivalent. As $H \to 0$, the solid phase ceases to affect the thermal field of the fluid, which is free to act independently, while as $H \to \infty$ the solid phase and fluid phase have identical temperatures and may be treated as a single phase. At intermediate values of H we observe that the critical wavenumber for stationary mode attains a maximum value and returns back to the LTE value. However, the oscillatory critical wavenumber decreases monotonically with H for intermediate values of H.

Fig. 8(b) indicates the variation of critical wavenumber for the oscillatory mode with H for different values of γ. As stated earlier for very small values of H the solid phase does not affect the onset criterion, and therefore the critical wavenumber a_c becomes independent of γ for small H. On the other hand for large values of H, the critical wavenumber a_c is a function of γ, since the stability criterion depends on the mean properties of the medium. We observe from this figure that for intermediate values of H the critical wavenumber for each of the stationary, oscillatory and finite amplitude modes decreases with increasing γ.

In Fig. 8(c) we display the effect of Lewis number on the critical wavenumber. This figure indicates that the critical wavenumber for both stationary and oscillatory modes increases with increase in Le while that for the finite amplitude motions decreases with Le. Further the effect of Le on finite amplitude critical wavenumber becomes less significant for large values of Le. The variation of critical wavenumber with H for different values of diffusivity ratio is shown in Fig. 8(d). From this figure it is found that the oscillatory critical wavenumber increases with the increasing values of α. However, the critical wavenumber for stationary and finite amplitude motions is independent of diffusivity ratio.

The variation of critical frequency ω_c^2 for the oscillatory mode with H for different parameter values is shown through

Figs. 9(a)–(d). It is clear from these figures that the critical frequency increases from a constant value, when H is very small to its maximum value and then with the further increase in H, it decreases back to another constant value, when H is large. The effect of Ra_S on the critical frequency is displayed in Fig. 9(a). It is found that the critical frequency increases with increase in the value of Ra_S. A similar effect on ω_c^2 has been observed in Fig. 9(c) with the Lewis number. In Fig. 9(b) the effect of porosity modified conductivity ratio γ on the critical frequency is displayed. Since for small H the solid phase ceases to affect the onset criteria we observe from these figures that ω_c^2 remains independent of γ for very small H. Also since in the very large H limit the stability criterion depends on the mean properties of medium, ω_c^2 depends on γ. It is observed from this figure that the critical frequency of oscillations decreases with increase in the value of γ. The effect of diffusivity ratio α on the critical frequency is revealed in Fig. 9(d). The effect similar to that of Ra_S and *Le* is observed for small values of H, while a reverse effect is observed for large values of H.

The variation of critical Rayleigh number, wavenumber and frequency of oscillations with H for different values of Vadasz number is unveiled in Figs. 10(a)–(c). From Fig. 10(a) it is reported that for small values of *Va* the double diffusive convection sets in through the finite amplitude motions. The oscillatory onset occurs for the large values of *Va* when H is small, however the finite amplitude motions overcome the oscillatory mode for moderate and large values of H. Therefore, the instability once again sets in through the finite amplitude mode. The critical Rayleigh number for oscillatory mode decreases with increase in *Va* whereas stationary and finite amplitude critical Rayleigh number remain independent of *Va*. The effect of Vadasz number is therefore, to advance the onset of double diffusive convection, in oscillatory mode.

In Fig. 10(b) we display the effect of Vadasz number on the oscillatory critical wavenumber. This figure indicates that the critical wavenumber a_c^{Osc} for oscillatory mode increases with *Va*. However, the there is no significant effect of *Va* on the stationary and finite amplitude critical wavenumber. The effect of *Va* on the critical frequency of oscillations is depicted in Fig. 10(c). The effect similar to that of Ra_S and *Le* is observed from this figure. That is the effect of *Va* is to increase the critical frequency.

In the study of double diffusive convection the determination of heat and mass transport across the layer plays a vital role. Here, the onset of convection as the thermal Rayleigh number is increased is more rapidly detected by its effect on the heat and mass transfer. The quantity of heat and mass transfer across the layer is given by *Nu* and *Sh* respectively, which represent the ratio of heat or mass transported across the layer to the heat and mass transported by conduction alone. In Figs. 11(a)–(d) we exhibit the variation of the Nusselt number of the fluid phase and the Sherwood number with thermal Rayleigh number for different values of Ra_S, H, *Le* and γ. From each of these figures it is clear that as Darcy–Rayleigh number increases from one to three times of its critical value, the heat and mass transfer increase sharply and as the thermal Rayleigh number is increased further, they remain almost constant. It is also found

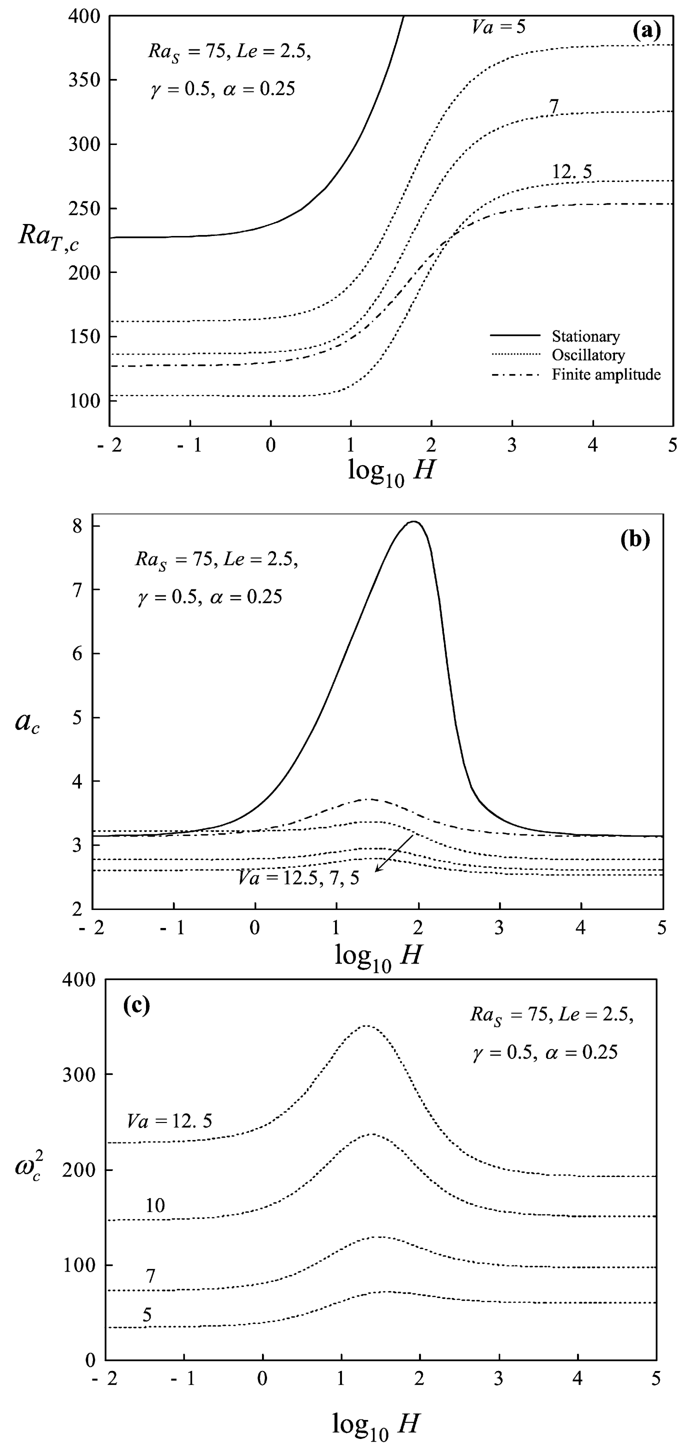

Fig. 10. Variation of critical values of (a) Rayleigh number (b) wavenumber and (c) frequency with H for different values of Vadasz number.

that in most of the cases the Sherwood number is above the Nusselt number. We also note that the effect of each of Ra_S, H, γ and *Le* is to increase the values of *Nu* and *Sh*. Therefore the effect of each of these parameters is to enhance the heat and mass transport across the layer. Although the presence of a stabilizing gradient of solute will inhibit the onset of convection, due to the strong finite amplitude motions, which exist for large Rayleigh numbers, tend to mix the solute and redistrib-

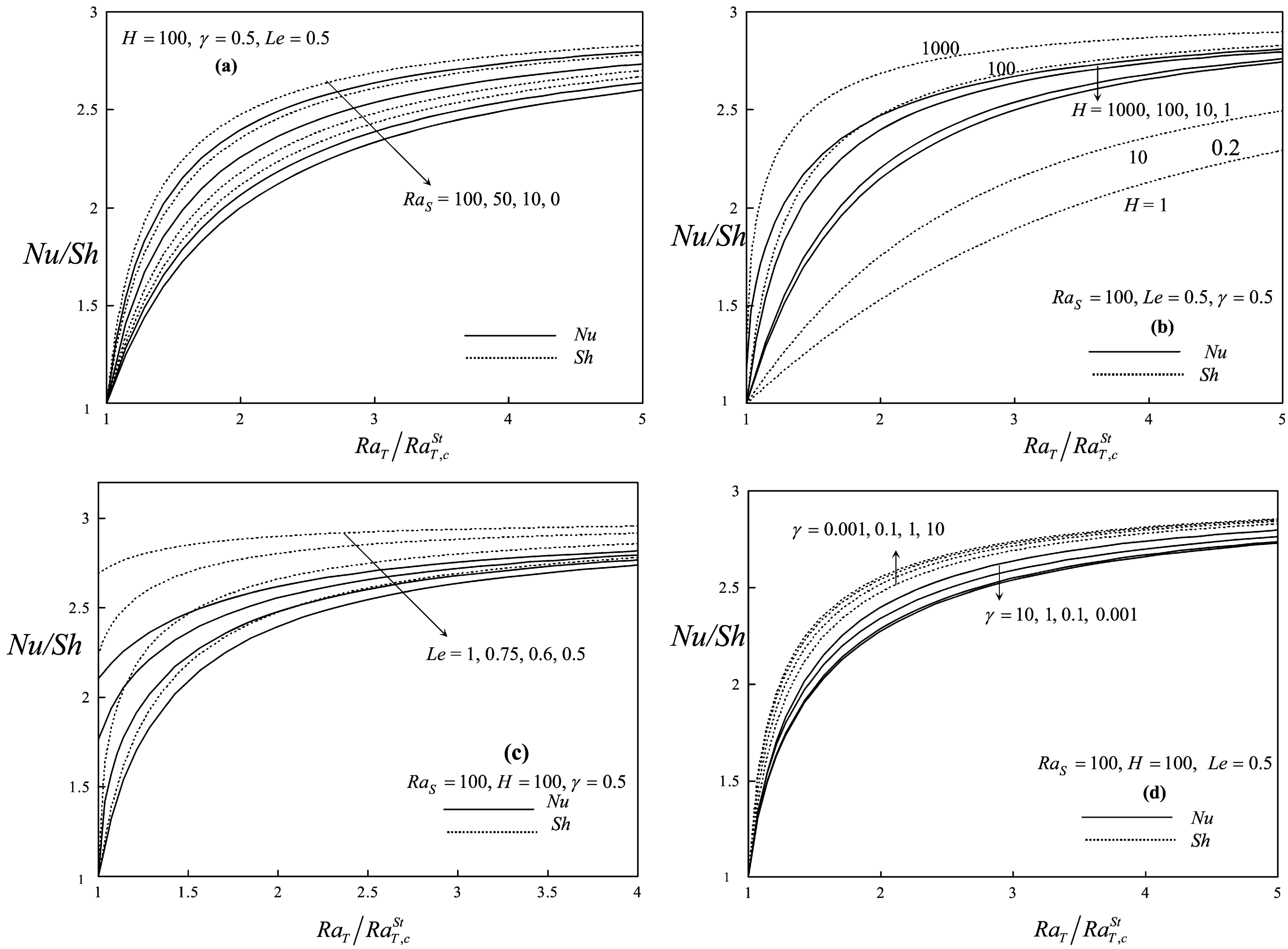

Fig. 11. Variation of Nusselt/Sherwood number with Rayleigh number Ra_T for different values of (a) Ra_S, (b) H, (c) Le and (d) γ.

ute it so that the interior layers are more neutrally stratified. As a consequence of that the inhibiting effect of solute gradient is greatly reduced and hence fluid will convect more and more heat and mass when there is an increase in the value Ra_S, H, γ and *Le*.

7. Conclusions

The linear and nonlinear double diffusive convection in a horizontal fluid-saturated porous layer is investigated analytically when the fluid and solid phases are not in local thermal equilibrium. In case of linear theory the thresholds of both stationary and oscillatory convection are derived as the functions of solute Rayleigh number, inter-phase heat transfer coefficient, Lewis number, porosity modified conductivity ratio, Vadasz number and diffusivity ratio. The nonlinear theory predicts the occurrence of finite amplitude motions. We found that there is competition between the processes of thermal and solute diffusion that causes the convective instability to set in as oscillatory and finite amplitude mode rather than stationary. It is found that for both large and small inter-phase heat transfer coefficient the system behaves like a LTE model while the intermediate values have strong influence on each of stationary, oscillatory and finite amplitude modes. The presence of a stabilizing gradient of solute will inhibit the onset of double diffusive convection. The effect of porosity modified conductivity ratio, Vadasz number is to enhance the instability of system. The Lewis number stabilizes the system towards the stationary mode while destabilizes the oscillatory and finite amplitude modes. The diffusivity ratio strengthens the stabilizing effect of inter-phase heat transfer coefficient. For very small values of H, the critical values are independent of the porosity modified conductivity ratio γ as there is almost no transfer of heat between the fluid and solid phases. Finite amplitude results are used to evaluate the convective heat and mass flux. The strong finite amplitude motions, tend to mix the solute and redistribute it so that the interior layers are more neutrally stratified. As a consequence of that the inhibiting effect of solute gradient is greatly reduced and hence fluid will convect more and more heat and mass when there is an increase in the value Ra_S, H, γ and *Le*.

Acknowledgements

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References

[1] D.A. Nield, A. Bejan, Convection in Porous Media, third ed., Springer-Verlag, New York, 2006.
[2] D.A. Nield, Onset of thermohaline convection in a porous medium, Water Resource Res. 4 (1968) 553–560.
[3] N. Rudraiah, P.K. Srimani, R. Friedrich, Finite amplitude convection in a two component fluid saturated porous layer, Int. J. Heat Mass Transfer 25 (1982) 715–722.
[4] D. Poulikakos, Double diffusive convection in a horizontally sparsely packed porous layer, Int. Commun. Heat Mass Transfer 13 (1986) 587–598.
[5] N. Rudraiah, M.S. Malashetty, The influence of coupled molecular diffusion on the double diffusive convection in a porous medium, ASME J. Heat Transfer 108 (1986) 872–876.
[6] J.W. Taunton, E.N. Lightfoot, T. Green, Thermohaline instability and salt fingers in a porous medium, Phys. Fluids 15 (1972) 748–753.
[7] M.E. Taslim, U. Narusawa, Binary fluid composition and double diffusive convection in porous medium, J. Heat Mass Transfer 108 (1986) 221–224.
[8] O.V. Trevisan, A. Bejan, Mass and heat transfer by natural convection in a vertical slot filled with porous medium, Int. J. Heat Mass Transfer 29 (1986) 403–415.
[9] B.T. Murray, C.F. Chen, Double diffusive convection in a porous medium, J. Fluid Mech. 201 (1989) 147–166.
[10] B. Straughan, K. Hutter, A priori bounds and structural stability for double diffusive convection incorporating the Soret effect, Proc. R. Soc. Lond. A 455 (1999) 767–777.
[11] A. Amahmid, M. Hasnaoui, M. Mamou, P. Vasseur, Double-diffusive parallel flow induced in a horizontal Brinkman porous layer subjected to constant heat and mass fluxes: analytical and numerical studies, Heat Mass Transfer 35 (1999) 409–421.
[12] M. Mamou, P. Vasseur, Thermosolutal bifurcation phenomena in porous enclosures subject to vertical temperature and concentration gradients, J. Fluid Mech. 395 (1999) 61–87.
[13] R. Bennacer, A. Tobbal, H. Beji, P. Vasseur, Double diffusive convection in a vertical enclosure filled with anisotropic porous media, Int. J. Thermal Sci. 40 (2001) 30–41.
[14] M. Mamou, P. Vasseur, M. Hasnaoui, On numerical stability analysis of double diffusive convection in confined enclosures, J. Fluid Mech. 433 (2001) 209–250.
[15] R. Bennacer, A. Tobbal, H. Beji, Convection naturelle thermosolutale dans une cavité poreuse anisotrope: Formulation de Darcy–Brinkman, Rev. Energ. Ren. 5 (2002) 1–21.
[16] A. Bahloul, N. Boutana, P. Vasseur, Double diffusive and Soret-induced convection in a shallow horizontal porous layer, J. Fluid Mech. 491 (2003) 325–352.
[17] A.A. Hill, Double-diffusive convection in a porous medium with a concentration based internal heat source, Proc. R. Soc. A 461 (2005) 561–574.
[18] R. Bennacer, A.A. Mohamad, M. El Ganaoui, Analytical and numerical investigation of double diffusion in thermally anisotropy multilayer porous medium, Heat Mass Transfer 41 (2005) 298–305.
[19] A. Mansour, A. Amahmid, M. Hasnaoui, M. Bourich, Multiplicity of solutions induced by thermosolutal convection in a square porous cavity heated from below and submitted to horizontal concentration gradient in the presence of Soret effect, Numer. Heat Transfer, Part A 49 (2006) 69–94.
[20] D.B. Ingham, I. Pop (Eds.), Transport Phenomena in Porous Media, Pergamon, Oxford, 1998.
[21] D.B. Ingham, I. Pop (Eds.), Transport Phenomena in Porous Media, vol. III, Elsevier, Oxford, 2005.
[22] A.V. Kuznetsov, K. Vafai, Analytical comparison and criteria for heat and mass transfer models in metal hydride packed beds, Int. J. Heat Mass Transfer 38 (1995) 2873–2884.
[23] A.V. Kuznetsov, A perturbation solution for a non-thermal equilibrium fluid flow through a three-dimensional sensible storage packed bed, Trans. ASME J. Heat Transfer 118 (1996) 508–510.
[24] K. Vafai, A. Amiri, Non-Darcian effects in combined forced convective flows, in: D.B. Ingham, I. Pop (Eds.), Transport Phenomenon in Porous Media, Pergamon, Oxford, 1998, pp. 313–329.
[25] A.V. Kuznetsov, Thermal non-equilibrium forced convection in porous media, in: D.B. Ingham, I. Pop (Eds.), Transport Phenomenon in Porous Media, Pergamon, Oxford, 1998, pp. 103–130.
[26] D.A.S. Rees, I. Pop, Local thermal non-equilibrium in porous medium convection, in: D.B. Ingham, I. Pop (Eds.), Transport Phenomena in Porous Media, vol. III, Elsevier, Oxford, 2005, pp. 147–173.
[27] D.A.S. Rees, I. Pop, Vertical free convective boundary layer flow in a porous medium using a thermal non-equilibrium model, J. Porous Media 3 (2000) 31–44.
[28] D.A.S. Rees, Vertical free convective boundary layer flow in a porous medium using a thermal non-equilibrium model: elliptic effects, Zeitschrift fur Angewandte Mathematik und Physik (ZAMP) 54 (2003) 437–448.
[29] N. Banu, D.A.S. Rees, Onset of Darcy–Benard convection using a thermal non-equilibrium model, Int. J. Heat Mass Transfer 45 (2002) 2221–2228.
[30] A.C. Baytas, I. Pop, Free convection in a square porous cavity using a thermal non-equilibrium model, Int. J. Thermal Sci. 41 (2002) 861–870.
[31] A.C. Baytas, Thermal non-equilibrium natural convection in a square enclosure filled with a heat-generating solid phase non-Darcy porous medium, Int. J. Energy Res. 27 (2003) 975–988.
[32] A.C. Baytas, Thermal non-equilibrium free convection in a cavity filled with a non-Darcy porous medium, in: D.B. Ingham, A. Bejan, E. Mamut, I. Pop (Eds.), Emerging Technologies and Techniques in Porous Media, Kluwer Academic, Dordrecht, 2004, pp. 247–258.
[33] N.H. Saeid, Analysis of mixed convection in a vertical porous layer using non-equilibrium model, Int. J. Heat Mass Transfer 47 (2004) 5619–5627.
[34] M.S. Malashetty, I.S. Shivakumara, K. Sridhar, The onset of Lapwood–Brinkman convection using a thermal non-equilibrium model, Int. J. Heat Mass Transfer 48 (2005) 1155–1163.
[35] M.S. Malashetty, I.S. Shivakumara, K. Sridhar, The onset of convection in an anisotropic porous layer using a thermal non-equilibrium model, Transport in Porous Media 60 (2005) 199–215.
[36] B. Straughan, Global non-linear stability in porous convection with a thermal non-equilibrium model, Proc. Roy. Soc. Lond. A 462 (2006) 409–418.
[37] P. Vadasz, Explicit conditions for local thermal equilibrium in porous media heat conduction, Transport in Porous Media 59 (2005) 341–355.
[38] P. Vadasz, Coriolis effect on gravity-driven convection in a rotating porous layer heated from below, J. Fluid Mech. 376 (1998) 351–375.
[39] B. Straughan, A sharp nonlinear stability threshold in rotating porous convection, Proc. Roy. Soc. Lond. A 457 (2001) 87–93.
[40] P. Vadasz, S. Olek, Weak turbulence and chaos for low Prandtl number gravity driven convection in a porous media, Transport in Porous Media 37 (1999) 69–91.
[41] P. Vadasz, Local and global transitions to chaos and hysteresis in a porous layer heated from below, Transport in Porous Media 37 (1999) 213–245.
[42] P. Vadasz, S. Olek, Route to chaos for moderate Prandtl number convection in a porous layer heated from below, Transport in Porous Media 41 (2000) 211–239.
[43] C.W. Horton, F.T. Rogers, Convection currents in a porous medium, J. Appl. Phys. 16 (1945) 367–370.
[44] E.R. Lapwood, Convection of a fluid in a porous medium, Proc. Cambridge Phil. Soc. 44 (1948) 508–521.
[45] P. Vadasz, Heat transfer regimes and hysteresis in porous media convection, ASME J. Heat Transfer 123 (2001) 145–156.